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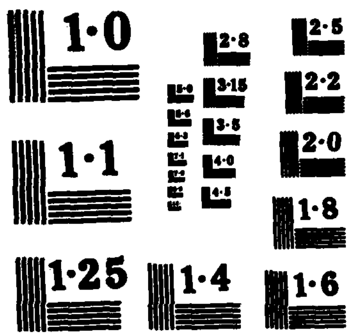

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INFORMATION SYSTEMS LABORATORY

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ALGORITHMS FOR LOCATING AND IDENTIFYING MULTIPLE SOURCES AND RECEIVERS BY A DISTRIBUTED SENSOR NETWORK

Technical Summary Report to the

Defense Advanced Research Projects Agency

Contract Number: MDA903-80-C-0331
Reporting Period: 16 December 1978 - 15 December 1980
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ABSTRACT

This report summarizes the research activities at the Information Systems Laboratory, Stanford University, for the Distributed Sensor Network project during the past two years. The objective of this research effort was to develop new signal processing techniques for locating, tracking and classifying multiple sources and receivers in a DSN system. The major accomplishments presented are in the areas of: source location techniques, estimation algorithms for DSN, and the development of a signal processing workbench. A comprehensive overview of our research results is presented with emphasis on their applications to various facets of the DSN problem.

TABLE OF CONTENTS

1. INTRODUCTION

This report summarizes the research activities in the Information Systems Laboratory (ISL) at Stanford University for the Distributed Sensor Network (DSN) project. Our effort is part of a DARPA sponsored program with several participant contractors. This report covers the period 16 December 1978 to 15 December 1980.

Our research effort during this reporting period concentrated on three principal areas:

- source location techniques
- estimation algorithms for DSN
- development of a signal processing workbench.

This work continues and enhances the research performed during the first year of the project, as reported in [DSN]. Our major accomplishments are briefly summarized below.

Source Location Techniques

Our original formulation of the problem of estimating source location from time difference-of-arrival, as a (conditionally) linear problem, proved to be very useful. It enabled us to develop for the first time optimal location estimation algorithms and to perform a detailed analysis of the identifiability issue. The results of this analysis were new insights into the sensor placement problem (i.e., what is the best configuration for a given set of sensors from the standpoint of location estimation accuracy) and the development of efficient location estimation algorithms.

We continued to develop our ARMA modeling approach to the multiple source location problem. Some of the key theoretical issues related to uniqueness were addressed and partially resolved, and a specific algorithm was developed and tested. This approach is a first attempt to handle simultaneously several sources, rather than perform single source detection and localization and then be faced with the association problem.

Several advances were made in the area of applying image reconstruction techniques to the multiple source location problem. An efficient minimum variance estimator was developed, which provides a key step towards a practical implementation of such algorithms in a DSN environment. Solutions were obtained for the problem of joint estimation of source distribution and attenuation. This problem is central to the application of reconstruction techniques to multi-source tracking by acoustic arrays.

Estimation Algorithms for DSN

A major achievement during the last reporting period was the development of ladder forms for time-series modeling and estimation. The simplicity and efficiency of ladder algorithms, which provide an exact least squares solution to the multichannel autoregressive modeling problem, is a significant breakthrough in the areas of estimation and adaptive signal processing. The theory and application of these ladder forms is expected to have a strong impact on the general field of signal processing and on the acoustic signal processing of the DSN type, in particular. A tremendous amount of interest was generated by our presentations of the recursive ladder forms, and the number of publications in this area seem to be increasing at an exponential rate. A byproduct of this was the development of a sensitive event detection technique for DSN applications, and its demonstration in a speech processing problem (in the context of pitch detection).

Another very significant achievement was the development of doubling algorithms for the efficient inversion of Toeplitz matrices. The need for inverting Toeplitz matrices arises in numerous problem areas. Our technique provides a significant reduction in the amount of computation needed to invert large matrices of this type. This development will have an impact on adaptive array processing and adaptive beamforming, for example. Both of these problems are of great importance to DSN systems. Other adaptive signal processing techniques that are needed in the uncertain environment in which the DSN operates were developed as part of our research effort.

Development Of A Signal Processing Workbench

An essential part of the process of developing new techniques for source parameter estimation is the testing of proposed algorithms by computer simulation. Simulations allow us to study various issues related to the performance of the algorithms and to gain insights into their behavior. Analysis alone is not adequate, especially when recursive stochastic algorithms are concerned. Simulations are an invaluable tool for studying issues such as convergence rates, robustness, and estimation accuracy. Therefore, establishment of a signal processing workbench that will allow easy comparison and modification of processing steps and data sequences is important. In particular, this will facilitate the exchange of processing tools and data bases among DSN researchers and maximize the impact of our research results on the design of a DSN system.

In the proposed workbench, the processing modules will function in a stand alone fashion with standard input/output formats such that complex signal processing functions are built by the simple cascading of modules. In the UNIX† operating system, precompiled stand alone modules can be connected together using a system command called a pipe. The pipe connects the output of one

† See *Bell System Technical Journal*, Vol. 57, No. 6, July-August 1978.

module to the input of the next. Since each module is a precompiled program, the signal processing can be changed quickly without recompiling or linking, just by forking to the appropriate module. Using the UNIX system debugger commands allows the internal parameters of a signal processing module to be observed and modified as data is being processed. This facility to monitor and change signal processing parameters while data is being processed enhances the debugging, development, and understanding of new algorithms. The ability to control multiple processes is not available in the debuggers provided with the current releases of the UNIX operating system. Some experience with the problem of adding this capability has been obtained on the DEC-11/34 through the development of our own debugger. This experience will be applied to extend the available debuggers on the VAX. We are continuing to develop a signal processing workbench based on the above ideas.

In addition to these main thrust areas, progress was made in our basic research into distributed alg[illegible]rithms. Interesting results were obtained on two dimensional systems, which have implications for planar sensor arrays of the type encountered in a DSN system. New efficient estimation algorithms were developed for nearly stationary processes. This development opens the way for relaxing the stationarity assumption that is commonly made in current techniques for location estimation and spectral estimation. The framework related to nearly stationary processes lends itself naturally to distributed implementation and is, therefore, well suited to DSN applications.

In the next phase of the project we plan to continue our investigation on two levels: basic research into fundamental mathematical questions related to DSN system design, and the development, coding, and testing of signal processing modules for a prototype DSN system. An important part of our work will be the testing of new signal processing techniques on data provided by Lincoln Laboratories. This will enable us to demonstrate the performance gains that can be achieved by more sophisticated processing of multi-sensor data in a DSN scenario.

The results of our research effort were summarized in numerous publications as indicated by the list in Section 7. The objective of this report is to present an overview of our work in a coherent framework, while emphasizing its significance to different aspects of the DSN problem. We, therefore, limit our technical discussions to brief descriptions of the main results, leaving most of the details to other publications. It is hoped that this report will provide an introduction and summary of the research performed at the ISL for the DSN project, enabling the reader to identify specific topics of interest and pointing to the relevant references for more details. For completeness we have included some of the key publications as appendices.

2. SOURCE LOCATION TECHNIQUES

l Overview

A central problem in a distributed sensor network is the determination of e presence and location of sources of acoustic and electromagnetic energy in e area under surveillance. Multiple sensors provide information which need to processed and integrated in order to obtain a picture of the outside world. me of the classical approaches to this problem are: high resolution spatial ectral estimation [CGK], beamforming for sensor arrays [VT], and Kalman tering techniques and various multitarget tracking techniques (see e.g., the rvey in [YBS]). Most of the work in this area was done in the context of a sin- e sensor (e.g., a radar system) or a group of sensors acting as a single sensor .g., a linear antenna array or a hydrophone array).

The DSN problem has certain features which do not seem to fit well these assical approaches. These include: the geographical dispersion of the sensors, e need for distributed processing (all the methods mentioned above use a mpletely centralized processing architecture), the presence of constraints on e amount of data that can be communicated from the sensor to the process- g nodes in the network, and the large number of sensors and sources that may e involved. In view of these differences we found it necessary to explore new chniques for solving the source location problem that seem better suited for SN applications, including *bistatic* and "*cooperative*" modes.

In our treatment of the source location problem we considered three main pproaches. The first involves the estimation of time-differences-of-arrival DOA) of the signal to several sensors, followed by estimating the location of the gnal source. This type of approach has been used in passive sonar systems. e looked much more closely at the problem structure and were able to develop some improved location estimation techniques (Section 2.2 and 2.3). Furthermore, this analysis led to a characterization of source identifiability, i.e., under what conditions is it possible to uniquely locate a source with a given set of measurements. This analysis provides some suggestions about the best way of placing sensors in a DSN system.

In Section 2.4 we show that the source location problem can be formulated as a system identification problem, where the system consists of several sources and sensors. This approach provides a novel way of simultaneously estimating the TDOA-s and spectral characteristics of multiple sources. The proposed technique involves multichannel time-series modeling. Efficient algorithms for performing such modeling were also developed, as will be discussed in Section 3. These algorithms are well-suited for distributed implementation and provide an attractive alternative to FFT related techniques.

In Section 2.5 we present a noncoherent signal processing approach for generating an image of the area, depicting the multiple sources that are present in it. This approach is based on the idea of image reconstruction from line integral projections, which has been used extensively in medical applications such as Computer Aided Tomography (CAT). We show that by treating the measurements of certain types of sensors as line integrals through an "energy intensity map", it is possible to reconstruct an image showing the location and intensity of various sources of acoustic/electromagnetic energy in a given area.

Finally, in Section 2.6 we discuss a technique for remote sensing of the atmosphere, to estimate propagation parameters. Knowledge of such parameters can be used to improve the accuracy of the location estimates.

urement geometry commonly employed in CT applications can lead to a dramatic reduction in the total computation required for at least one of these methods, the minimum variance estimator, thereby making it more competitive with the various approximate inversion techniques [WM]. In DSN applications, computational efficiency is crucial, because of the large amounts of data involved and the temporal resolution required.

Summary of [NUN]

As mentioned earlier, the acoustic imaging problem involves estimation of both the image of sources and the attenuation of the signals from the sources to the sensors. This problem was treated by Nunes [NUN] in the context of medical imaging. The following is a summary of the topics discussed in Nunes' thesis.

A usual approach to the reconstruction problem in computerized tomography consists of representing the image as a finite vector and the measurements as a linear operation on this vector plus an additive noise. This modeling approach has several advantages over Radon transform-based techniques in the sense that the measurement geometry is not critical, allowing good results when evenly spaced measurement profiles cannot be obtained; in addition, the measurement noise is part of the model, leading to the use of estimation techniques for designing reconstruction algorithms.

We considered three topics associated with this approach. First, the modeling of the measurements is studied when the elements of the image vector are the first n coefficients of the expansion of the three dimensional distribution to be reconstructed into a complete set of orthogonal functions. Then several algorithms that are currently used for transmission reconstruction are studied and new results are presented. Finally, the more complex problem of reconstructing an emission distribution embedded in a medium with an unknown absorption ıt first we present a brief summary of the relevant research results.

ummary of [WM]

In computerized tomography the number of x-ray photons transmitted ırough an object of interest is measured by an array of detectors. This array is ɔtated by a fixed angular increment between each set of measurements. From ıese measurements the total attenuation along each measurement path is erived and an image of the plane of the measurement paths may be reconructed. A detailed description of this application and several approaches to ɔlutions are discussed in [MAC], [GOR], [NS] and [CBM].

Exact deterministic methods for the reconstruction of an image of the ıeasurement plane can not be directly applied to computerized tomography for wo reasons. First, the measurements contain the inevitable measurement oise along with noise due to counting statistics. In addition, the measurements re not continuous, but are taken only at discrete intervals and angular posi.ons. Because of these complications, reconstructing an image is an estima.on, and not an inversion problem. Estimation theory results have been used in ıis application by Herman [HL], Rockmore [ROC], Wood [WOO], Fortes [FOR], nd Nunes [NUN], using Bayesian estimation, maximum likelihood estimation, ıinimum variance estimation, and maximum a posteriori estimators respec.vely. These methods generally produce results that are superior to the ones btained via approximate inversion techniques currently employed, especially in ases of low dosage and high measurement noise [ROC], [WOO]. There is growing ıterest in quantitative and automatic processing of CT images, and we contend hat statistically optimal images are more appropriate for this application.

The estimators discussed above are not generally used in practice because f the computational complexity and storage requirements. However the meas-

Thus if many radars are observing a given volume of space, their measurements represent surface integrals through the radar image of that volume. A convolve and backproject reconstruction scheme for 3-D surface projections is presented in [RDF]. In the case of a ranging radar that uses an omnidirectional antenna, the projections will be on the surface of a sphere. Techniques for reconstruction from spherical projections can be similarly derived.

A somewhat different formulation has to be used in the case of acoustic sensors. An acoustic array can form multiple beams in different spatial directions. The energy received at the array output, is the integrated acoustic energy in the beam volume. Thus, a system consisting of several acoustic arrays observing a given area, provides a set of line integral measurements, in a fan-beam geometry. These measurements can be used to reconstruct the acoustic energy map of the observed area, from which the source locations can be estimated. The situation is somewhat more complicated than a standard reconstruction problem, because of the R^2 attenuation (R = range) due to spreading loss.

Several aspects of the image reconstruction approach were addressed in our research program. The work of S. Wood [WM] treated the computational aspects of image reconstruction algorithms, and derived a fast implementation of the minimum variance estimator. The work by Nunes [NUN] and Fortes [FOR] addressed the joint estimation of emission and attenuation characteristics. These type of solutions can be used to locate sources in the presence of various types of propagation loss. Most of the work on image reconstruction deals with a two-dimensional situation. This work also extends the analysis to three dimensions, making it possible to address the problem of locating sources both in range/bearing and in altitude. The research mentioned above was done in the context of medical applications. However, this approach is very promising for solving multiple source location problems as discussed in the following section,

2.5 Image Reconstruction Techniques For Multiple Source Location

2.5.1 Introduction and Summary

A class of inverse problems which appears in many scientific and engineering applications is the reconstruction of a density function from projections. In Computer Assisted Tomography (CAT), x-rays or γ-rays are used for imaging the internal structure of the body. In radio-astronomy, the electromagnetic energy measured by antenna arrays is used to reconstruct the radio-brightness map of the celestial sphere. Various techniques have been developed for solving the image reconstruction problem and this area is by now fairly well developed [BD] [SCU] [GH]. In this section we will briefly indicate how image reconstruction techniques can be applied to the DSN problem. See also [FDR] for a more detailed discussion.

Let us consider, for example, a radar system observing isotropic point scatterers distributed in a volume of space. Typically, its *range resolution* will be much better than its azimuth or elevation resolution. The radar will measure the total energy of returns from a volume of space which is essentially a thin layer, see Figure 2.2.

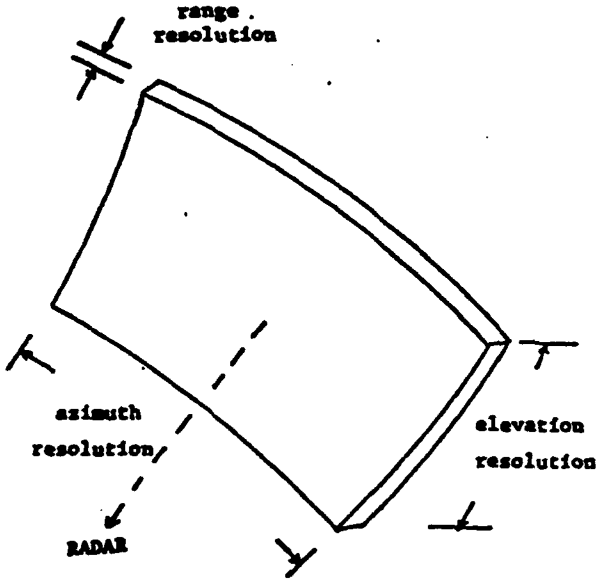


Figure 2.2 Surface Projection in Radar Measurements

original presentation of this approach we suggested the multichannel Recursive Maximum Likelihood technique. However, the development of computationally efficient and robust ladder algorithms, leads us to prefer them for actual implementation. Further motivation for this choice is some recent work which indicates that a version of the covariance ladder form can be used to go from Left-to-Right-MFD, a crucial step in our tracking technique. Furthermore, the ladder algorithms seem to be well suited for distributed implementation, and thus for application in a DSN system.

earthquake localization, intruder detection or artillery localization). The active tracking problem (radar, active sonar) can also be handled in this framework.

The proposed approach is based on the idea of fitting a multi-input multi-output model to the vector time-series observed at the outputs of the sensors. Under certain assumptions the parameters of this model can be shown to contain information about the locations of all the sources, as well as other information such as source spectra. If the parameter estimation is performed recursively and continually, it is hoped that the relationship between the model parameters and the sources to which they correspond will be consistently maintained. Once the sources are labeled in a particular way, this labeling will stay fixed without need for rechecking or relabeling. This will eliminate the need for a separate step of association, which is inherent in current multi-source tracking techniques.

The formulation of the tracking problem as a multichannel ARMA modeling problem was presented in [FM1] and will not be repeated here. We note, however, that the formulation of source tracking as a multichannel signal modeling problem raises a number of interesting questions in the areas of system identification and structure of multivariable linear systems. These questions include:

How to find a unique (not necessarily minimum phase!) spectral factor, under structural constraints;

How to estimate directly the parameters of a right Matrix Fraction Description from a given data set;

How to include certain structural constraints into the multichannel system identification problem.

Various algorithms can be used to estimate the ARMA parameters. In our

2.4 An ARMA Modeling Approach to Locating Multiple Sources [FM1]

Tracking multiple sources represents special difficulties since there can be uncertainties associated with the measurements beyond their inaccuracy, which is usually modeled by some additive noise. This additional uncertainty is related to the origin of the measurements. Since several sources are present, it is necessary to sort out which measurement corresponds to which source. In other words, in addition to the problem of detection and bearing/range estimation, there is a problem of properly labeling the set of measurements. The latter problem is usually referred to as source association or track formation.

Typically, these two facets of multi-source tracking are treated separately. First, a set of potential source locations is obtained. Then some method is used to label these locations by the sources to which they correspond in a manner consistent with previous measurements. Techniques for labeling or multi-source tracking have been developed using various approaches including: Kalman filtering, Bayesian methods, Integer programming, and Track splitting.

In all of these techniques the basic detection and location estimation are performed *separately* for each source. The multi-source aspect of the problem enters only through the labeling procedure. In other words, the tracking problem is treated as a collection of single source problems, which has to be put together in a systematic and consistent way.

In [FM1], we attempt to tackle directly the multichannel nature of the problem. Instead of looking at one source at a time, we want to estimate *simultaneously* multi-source parameters (such as location and spectrum). The approach is best understood in the context of the passive tracking, where an array of sensors measures signals (electromagnetic, acoustic or seismic) generated by sources. This type of problem arises in sonar systems, acoustic surveillance systems (detection of low flying aircraft) and seismic arrays (oil exploration, Gaussian second order filter developed by Sorenson and Stubbard [SS] using some additional assumptions on the structure of the measurement update equations. Our derivation is more general in that it does not make any a priori assumptions regarding the structure of the estimator. The second order Gaussian filter is known to have better convergence properties than the EKF and is, therefore, preferable for tracking applications.

The second order Gaussian filter lends itself to distributed implementation. Several distributed forms of the filter equations are derived in [LM3]. A comparison with a completely centralized scheme demonstrates that our distributed algorithms lead to considerable savings in communication requirements. Moreover, they exhibit a parallel structure and are flexible. We believe that these algorithms will provide a practical intermediate solution for tracking in a DSN system. However, a solution based on the conditionally linear formulation earlier should be considered as the ultimate goal.

$$x_{k+1} = F_k x_k + \sum_{i=1}^{n} e^i x_k' G_k^i x_k + w_k \qquad (2.3)$$

where $\{e^1, e^2, \ldots, e^n\}$ is the standard basis of $\mathbf{R}^n$ and the matrices G_k^i are symmetric.

The presence of a quadratic measurement equation leads to a nonlinear estimation technique which is typically solved by using an Extended Kalman Filter (EKF) [JAZ]. The EKF is based on a successive linearization of the measurement and state equations. While the EKF seems to work well in some tracking situations, various authors report encountering serious difficulties especially in near range and low signal-to-noise situations. A principal drawback of the EKF is that it is not guaranteed to converge. Divergence of the estimation error is fairly common, especially under the conditions mentioned above.

The problem formulation we presented in Section 2.2 and in [LM4], opens the way for alleviating these difficulties. In our approach the measurement equations are linear (or at least conditionally linear). If in addition, the source dynamics are assumed to be also linear (a common assumption), the problem can be solved using the standard Kalman filter! This circumvents the need for any linearizations and the associated inaccuracies and divergence problems. The combination of our static source location technique with a linear Kalman filter has the potential of providing an elegant solution to the tracking problem. We should note, however, that this method is not yet fully developed and we are still working on some of the issues that need to be resolved before a practical application can be attempted.

Another way of alleviating divergence problems is to develop an alternative nonlinear estimation technique. Under the assumption of a Gaussian a-posterior density function, we obtained in [LM3] a finite order solution that does not involve linearization. Our estimation algorithm turns out to be identical with the

2.3 Distributed Estimators For Quadratic Systems [LM3]

The estimation technique described in the previous section was presented as the solution to a static problem: given a set of TDOA measurements corresponding to a certain source location, find the best estimate of that location. The source tracking problem involves, however, a dynamic aspect that is not captured by the discussion presented earlier. In this section we present an approach that addresses the time-varying features of tracking one or more sources.

A standard approach to the tracking problem is to construct a nonlinear dynamic equation that describes the evolution of the source state vector. This state vector will typically contain the source position coordinates (two or three variables), its velocity and perhaps the clock bias. See for example, the discussions in [NM] - [DMF1]. The measurement equation can be shown to exhibit on intrinsic quadratic structure

$$z_k = H_k x_k + \vec{1}\, x_k' E x_k + v_k \; , \tag{2.1}$$

where

$$\vec{1} = [1, 1, \ldots, 1]' \times R^p$$

$$u_k = \text{white noise process}$$

The measurement matrix H_k may be time-varying, when the sensors themselves are moving. The state vector typically obeys a linear evolution equation of the type

$$x_{k+1} = F_k x_k + w_k \; . \tag{2.2}$$

where w_k is a white noise process, and F_k is a matrix representing source motion dynamics. In fact, the approach can handle with no significant increase of complexity nonlinear dynamics of a quadratic type, i.e.,

[LM3].

The use of a two step procedure, i.e., involving a preprocessing step, is very general for this estimation problem. In [SCH] a TDOA "averaging" procedure was proposed and a straightforward generalization of the idea is to estimate from the "raw" or measured TDOA's via linear least squares techniques the ranges up to an arbitrary constant, that can be viewed as a clock bias; a *consistent* set of TDOA's is directly obtained from these estimates. Since more structural information is used in our preprocessing step than in a simple TDOA averaging step, the corresponding estimates are in general more accurate. The overall estimation procedure also performs in general better than more traditional approaches, especially since the equation coefficients are measurement independent.

In [DMF1] the convenient approach, which separates the TDOA estimation and the source location problems has been followed. However, it is clear that a completely satisfactory answer to both the identifiability and estimation problems will only result from a global approach. A close combination of the approaches of [DMF1] [DMF2] and [FM1] appears to be possible and we are continuing our investigation in that direction.

[FM1] and [FR1].

In [DMF2] we first determine, as described above, the joint source and station geometries for which ambiguities in the determination of the source location arise. Then, using this result, we are able to introduce a simple rule for the placement of the stations to ensure that the source coordinates are always identifiable. It turns out that a linear array does not satisfy that rule and indeed the sensitivity of any source location estimate (geometric dilution of precision) is very high when the source lies close to the axis of the array. In the remainder of the paper the geometry of the stations is assumed to satisfy the placement rule. Under this mild condition our pxp matrix has additional properties that allow us to devise several simple estimation procedures.

Our estimation procedures involve two distinct steps and, assuming information from the second step is not fed back into the first step, are therefore suboptimal; their advantage lie in their relative simplicity. The first step, best viewed as a preprocessing step, consists of the (linear least squares) estimation of the ranges between the source and the stations. In [DMF1] a simple function of the ranges was estimated rather than the ranges themselves. This estimate was more robust but led to a more complex second step. Note that, at this level, the range estimates could be improved if some direct range measurements are available. In the second step the pxp matrix is factored using this range information; this yields a set of linear and quadratic equations in the source coordinates that can be viewed as our new or refined observations, replacing the TDOA's. The coefficients appearing in these equations are not a function of noisy measurements, they are functions of the stations coordinates *only*. Because this property is satisfied, standard estimation procedures can be applied to these equations, in the static case we can use for instance the results in [HB], or if a dynamic model is available, a second-order filter can be used, see e.g., A reasonable way to resolve such a difficulty is to reduce the original equations to an "equivalent" set of equations for which known estimation procedures are more easily applied. "Equivalent" means that the mapping from one set of equations to the other is one-to-one, i.e., no information is lost through that mapping. R. Schmidt made an interesting attempt in that direction in [SCH]. However, we have shown in [DMF1] that the $\binom{p}{2}$ linear equations he obtained are highly redundant and contain less information than the original TDOA's.

Simple notions of spatial rotation and translation invariance led us to consider a particular $p \times p$ matrix. The (i,j)-th entry is a function of the distance and range difference between the (i,j)-th pair of stations and has a simple expression in terms of the stations and source positions. That matrix equation plus some auxiliary sign information are easily shown to be equivalent to the original set of range differences.

An immediate advantage of rewriting of the TDOA's equations is that they can be analysed systematically using the simple tools of linear algebra. The cases for which, given the stations position and (noise free) range differences, the equations have more than one solution are completely characterized in the 2- and 3-dimensional Euclidean spaces. Thus the static "deterministic" identifiability problem is solved. This can be viewed as a first step in the solution of the "stochastic" identifiability problem, i.e., given noisy measurements, for both the static and moving source cases. Such a study is of fundamental interest since it indicates how much information about the source location can be extracted from the TDOA's and therefore the limitations of an estimation procedure. We carried out such a study in [DMF2]. The stochastic identifiability problem is discussed there along the lines of [PIC]. Ways to extend the approach to multi-source and multi-path situations are also indicated; however, the detailed study of these cases is not carried out in this paper and we refer to

2.2 Source Location From Time Differences of Arrival [DMF1] [DMF2]

The problem of source location from time differences of arrival (TDOA) is a standard nonlinear estimation problem. The usual approach to source location estimation is based on the construction of hyperbolic lines of position (LOP). Each measurement of TDOA at a pair of stations (or after multiplication by the velocity of propagation, each range difference) determines an LOP that is a branch of a hyperbola. Then the source location may be inferred from the intersections of the $\binom{p}{2}$ LOP's obtained from p stations. See Figure 2.1 for the case of 3 stations.

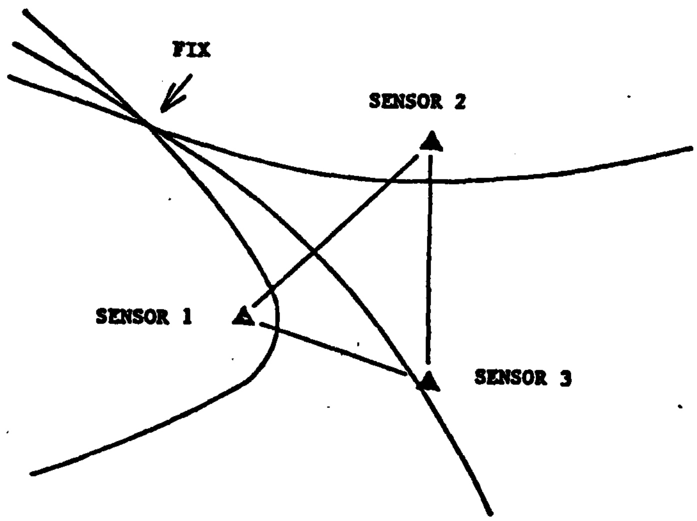


Figure 2.1. Hyperbolic Lines of Position

Unfortunately, information about the measurement noise cannot be completely accounted for in this approach, so that it seems impossible to claim some optimality for any estimator, or to characterize the statistical properties of the associated estimate.

coefficient distribution is treated as a joint estimation problem where the measurement equation is no longer linear.

The first part of this thesis develops an expression for transmission measurements that takes into account the quantization noise and the mathematical approximations necessary for the linearization of the measurement equation. Equations for emission and absorption measurements are also derived in this part.

Considering transmission reconstruction techniques, it is shown that two recently proposed algorithms asymptotically converge to the weighted least square solution. It is also shown that a priori information about the image vector can be included in the measurement set in order to make the algorithms converge to the linear minimum variance solution. An additional result is that the well known Algebraic Reconstruction Technique (ART) converges to a weighted least squares solution if a suitable time varying relaxation coefficient is used. We also show that convergence is achieved even if the set of measurements is not consistent.

Regarding emission reconstruction problems in the presence of unknown absorption, a nonlinear algorithm for jointly estimating the two distributions is developed, leading to the establishment of an estimation criterion which takes into account a priori information about the image. Additional algorithms are proposed and it is shown that they converge to a solution minimizing this new criterion.

Computer simulations of the transmission and joint estimation algorithms are presented, permitting the evaluation of their performance as a function of a limited number of iterations.

Summary of [FOR]

In a related work, Fortes [FOR] develops an estimation approach to 3-D reconstruction problems involving counting statistics, with applications to medical imaging. The following is a summary of the topics discussed in Fortes' thesis.

The problem of estimating three dimensional distributions from a given set of measurements that are noisy functionals of these distributions is analysed. The solution to this problem depends on several factors, but mainly on the measurement model and geometry.

In many applications measurement noise has a counting nature that allows a discrete stochastic process characterization. Hence, a general model assuming a Doubly Stochastic Poisson Process characterization for the measurement functions is presented. Based on this model, an estimation approach leading to the Maximum A Posteriori probability (MAP) estimate is carried out.

The presented formulation is clearly applicable to a large family of three dimensional reconstruction problems. However, to illustrate our results, the medical imaging problem of Computerized Tomography is mainly used. Computer simulations show that in transmission tomography our approach leads to a non-linear estimator that represents an improvement over the linear minimum mean squared error estimate. In particular, the squared error obtained with the MAP estimate is shown to have a lower mean and a narrower distribution than the one obtained with the Kalman Filter. In the case of emission tomography with unknown attenuation, simultaneous estimation of source and attenuation distributions is obtained using the MAP technique. In both cases, the performance of the MAP estimator is studied through a comparison with the Cramer-Rao mean squared error lower bound.

The necessary conditions for uniqueness of the MAP estimate for transmission and emission tomography with known attenuation are determined and shown to be related to the non-singularity of the projection matrices.

2.5.2 Image Reconstruction Techniques Applied to Target Detection

In Medical Imaging and Radioastronomy the images are generally "compact" and the ratio between the required resolution and the size of the image is not excessive, in the target detection problem the images are very sparse and the resolution required very large. This means that an excessive number of basis functions will be necessary in order to discretize the image. If the usual medical image reconstruction techniques are applied directly to the target detection problem, a colossal problem in terms of memory, computational time and number of required measurements will result. If the procedures are modified to consider the sparseness and positiveness of the images then medical imaging reconstruction techniques become efficient and valuable tools for solving of the multi-target detection problem. There are at least two ways of making medical image reconstruction techniques suitable for multi-target detection: pre-processing the data in order to reduce the actual image to a set of regions which are likely to contain targets, or dividing the image in pixels much greater than the expected size of a target and estimating the total target intensity for each pixel.

In medical imaging the image is generally divided in n adjacent nonoverlapping pixels, which can be seen as the first n basis functions. If the number of pixels is large enough, the quantization noise can be neglected. Of course the target detection problem can be modeled the same way; however, considering that the size of a target is very small compared to the region under surveillance, a very large number of pixels will be required in order to keep the quantization noise down. This implies a large amount of memory and computation time. For linear reconstruction techniques, at least as many measurements as the number of pixels should be used in order to have a unique solution, using the least-squares criterion for instance. One can argue that the linear minimum variance solution will always be unique. On the other hand, one can not expect good results when the number of measurements is small because essentially only second order prior statistics are used to complement the measurement set, but second order priors cannot represent two major a priori characteristics of the image: positiveness and sparseness.

Of course optimal nonlinear estimation criteria, like M.M.S.E. or M.A.P., can account for positiveness and/or sparseness. Unfortunately, if the pixel representation is kept, the amount of memory and computation time is still prohibitive at the moment without special purpose VLSI hardware. On the other hand, the sparseness and positiveness of the image intuitively suggest the use of nonlinear procedures as a preliminary processing of the image. This pre-processing would determine which pixels can be assumed empty and exclude them of further processing. Linear techniques can then be used for processing the remaining pixels. The pre-processing can also provide a priori statistics for the linear processing.

A data pre-processing procedure has been simulated that reduces the dimensionality of the multi-target detection problem. The method uses the sparseness and positiveness characteristics of the image for establishing "apriori" mean and covariance for the final processor. Since the image is positive and sparse, it is reasonable to assume that a measurement close to zero is nonzero only because there is noise involved, and all the pixels combined to form that measurement can be set to zero. Whether a measurement should be considered zero or not is decided by comparing it with a threshold that is a function of the noise level and of the other measurements. If a given pixel contributes to nonzero measurements in all profiles, then we assign to it a mean and a variance which give the Kalman filter (processor) a great flexibility in determining its value. If a pixel contributes to nonzero measurements in all but one profile, then we set to it a smaller mean than before and a smaller variance, i.e. we "tell" the processor that those pixels should be small.

The detection of targets using pre-processing as described above has been computer simulated. The model for the true signal point sources of differing intensity (referred to as phantom) and the geometry is illustrated in Figure 2.3. The additive noise was assumed constant. Figure 2.4 illustrates the lack of noise suppression in the backprojected reconstruction. The preprocessing with different thresholds is illustrated in Figure 2.5. The number and amplitude of the spurious sources increases as the threshold increases.

Another approach to the target detection problem is to redefine our objectives. Suppose that, instead of trying to determine the exact position of a target, we divide the region under observation in several pixels, each of them much bigger than the expected size of a target. We now want to determine the integral of the image in each pixel, instead of the image itself. Of course this set of parameters give us information about the distribution of targets in the area. This information could be sufficient in many cases. In this case a measurement is not an operation over the parameter space itself, but over the target distribution image. Of course it is possible that a linear operation over the image can be also represented as a linear operation over the parameter space. Loosely speaking, if all points of a pixel are treated in the same manner, they can be grouped together and replaced by the value of the integral over the pixel. In practice, it is not possible to treat all the points in a pixel in the same way. However in many cases we can say that they are treated *approximately* in the same way, which yields an approximated expression for a measurement in terms of the parameter space only.

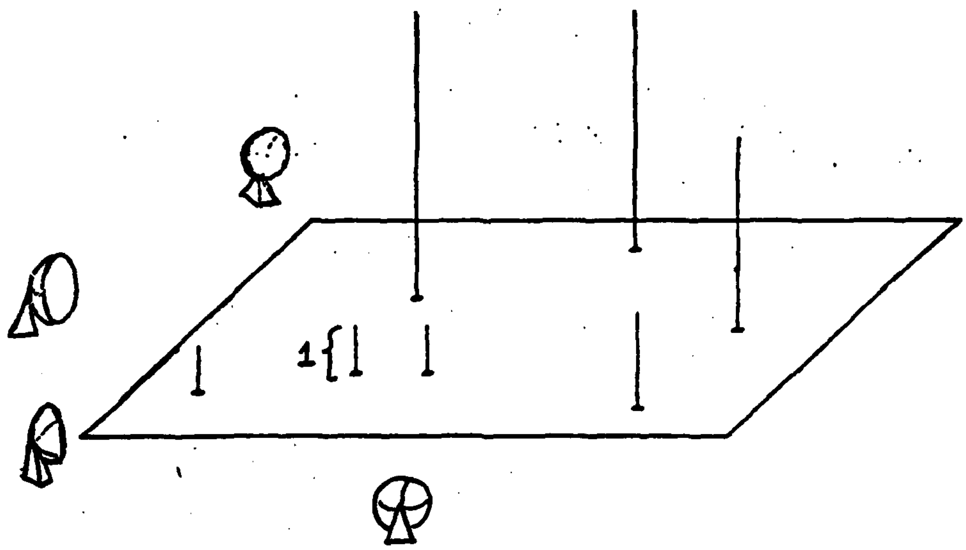


Figure 2.3 Source Location and Sensor Geometry

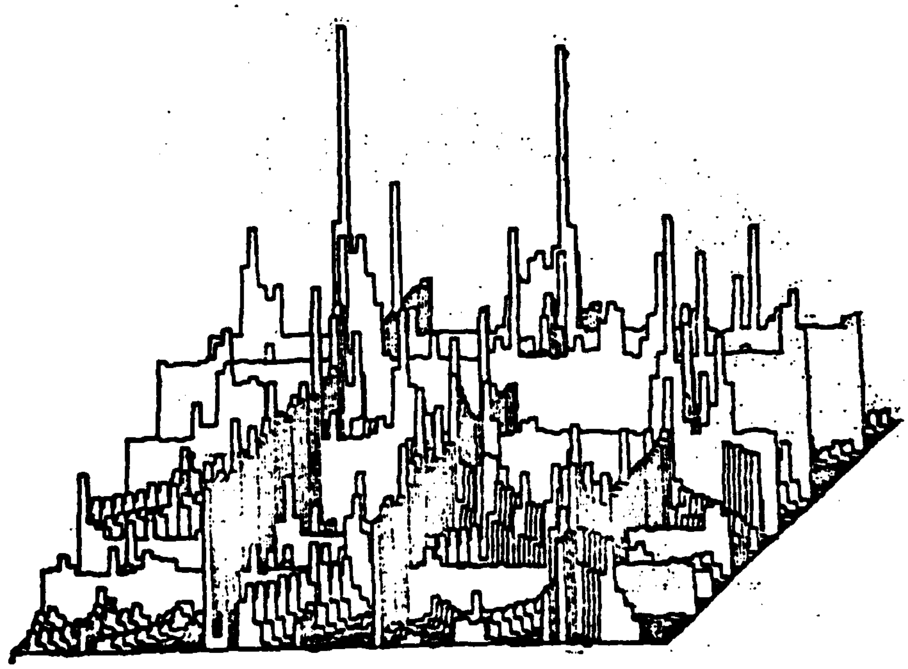
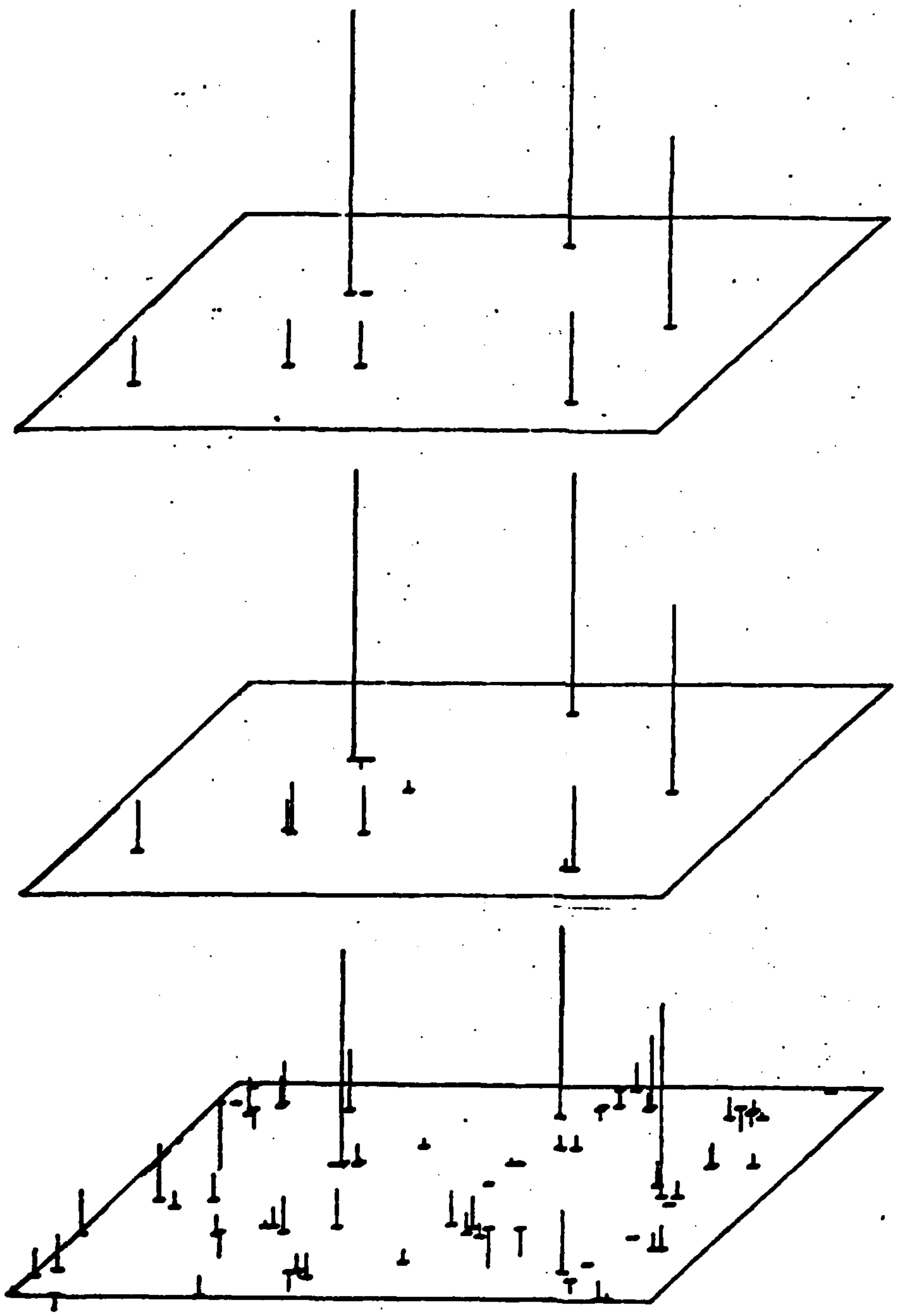


Figure 2.4 Backprojected Reconstruction

a) $\sigma_n^2 = .001$, b) $\sigma_n^2 = .01$, c) $\sigma_n^2 = .03$



a)

b)

c)

Figure 2.5 Pre-processing and Kalman Filtered Reconstruction

2.6 Optical Probing of the Atmosphere

The DSN concept is not limited to any particular class of sensors. However, the demonstration system designed by the Lincoln Lab group will make use of small acoustic arrays to detect and track low flying aircraft. The performance of such a system will depend among other things on the properties of the propagation medium, i.e., the atmosphere. Knowledge of various parameters such as temperature profile and wind velocity can improve location accuracy, especially at longer ranges. Working on a different problem, S. Delateur [DEL] investigated the possibility of estimating various atmospheric variables based on an optical probing system. The remote sensing techniques developed in his thesis may be applicable to an advanced DSN system. We next summarize the work described in [DEL].

When an optical wave travels through clear air turbulence, inhomogeneties in the index of refraction cause intensity variations at a receiving or imaging plane. These effects allow remote sensing of the earth's atmosphere. Two measurements of interest are path profiles of turbulence strength and wind velocity.

The lack of path selectivity and difficulties in signal processing (due to the non-stationarity of the atmosphere) have characterized methods which employ covariance calculations of the received field to estimate cross-path wind velocities. With a single-scatter, phase-screen model for clear air, an analysis of line-of-sight propagation was presented using Fourier optics, including a discussion of the assumed nature of the turbulence spatial spectrum. Specifications are detailed for the transmitter and receiver apertures and the processing of the received signal which provide estimates of the turbulence strength and transverse wind speed at selected points along the path.

An incoherent optical system incorporating these principles has been constructed. Results from experiments to monitor the movement of the atmosphere simultaneously at four turbulent scale sizes are reviewed with direct comparison between optical measurements of wind speed and local anemometer readings included.

Using computer simulations of the received signal characteristics, the system problems concerning velocity measurement errors and path profile selectivity were explored. Final comments contain improved signal processing techniques and aperture specifications for future equipment.

3. ESTIMATION ALGORITHMS FOR DSN

3.1 Overview

The problem of locating and classifying multiple sources on the basis of information received by different sensors in the network requires the use of various signal processing modules. Many of these modules embody some kind of an estimation algorithm. The performance of the DSN system will depend strongly on the quality of its "front end", i.e., the part of the system that takes raw sensor information and converts it into higher level parameters such as TDOA estimates, spectral estimates, or likelihood functions. Therefore, the development of estimation algorithms that are efficient and suitable for distributed implementation is an essential step towards the achievement of a high performance DSN system. Since multiple sensors and sensor arrays are involved, we need in general to consider multichannel estimation algorithms.

A major step in this direction is provided by our development of ladder forms for time-series modeling and estimation. This development can be considered as a significant breakthrough in the areas of least-squares estimation and adaptive signal processing. The body of theory and the numerous algorithms developed during the last reporting period will have a strong impact in many areas, including: speech processing, acoustic signal processing for sonar, adaptive arrays, and high resolution spectral estimation. A tremendous amount of interest was generated by our presentations of the theory and practice of ladder forms. The number of publications and applications related to ladder forms seems to be on an exponential growth curve. The highlights of these very exciting results are presented in Section 3.2. We believe that estimation techniques based on these ladder forms will lead to significant improvements in DSN performance, over that achievable with the more traditional techniques.

An interesting byproduct of the ladder form was the fact that one of the quantities being computed as part of their recursions is a sensitive indicator of events in the observed data. This led us to develop efficient algorithms that can be used to detect the appearance or disappearance of sources, or other events in a DSN scenario. In Section 3.3 we summarize some of the results in this area.

Another very important development in the area of efficient computational techniques was our work on doubling and tree algorithms. One main result of this work was the development of an inversion algorithm for NxN Toeplitz matrices that requires $O(N\log^2 N)$ operations. The need for inverting Toeplitz matrices arises in most estimation problems related to stationary, or nearly stationary (finite rank) processes. For general matrices we developed an efficient distributed algorithm, that requires only *local communications* between processors either arranged in a one or two dimensional array. An application of particular interest to the DSN problem is adaptive beamforming. Section 3.4 presents a brief description of our work in this area.

Ladder forms and other least-squares estimation techniques have numerous applications to adaptive signal processing problems. We investigated several such problems including adaptive line enhancement and adaptive linear phase filtering. Section 3.5 presents these and other related results.

3.2 Ladder Algorithms For Estimation and Adaptive Processing

A large number of publications were written on the theory and application of ladder forms during the last reporting period, e.g., [LEE] [LFM] [LM1] [FR2] [FR3] [PFM]. The number and diversity of research topics that were addressed makes it difficult to present a comprehensive summary. In this section we will therefore discuss only the highlights of our work. We start by introducing ladder structures and their properties.

Ladder Structures

Finite dimensional linear systems can be realized using different network configurations. These are typically classified as direct, cascade and parallel forms [OS]. As an example, consider an auto-regressive moving-average (ARMA) model given by

$$y_T = -\sum_{i=1}^{n} a_i \, y_{T-i} + \sum_{i=1}^{n} b_i \, u_{T-i} \, . \tag{3.1}$$

A direction realization of this model is depicted in Figure 3.1.

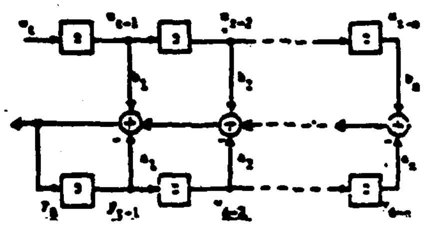


Figure 3.1. Direct Realization of $H(z) = B(z)/A(z)$

Many other direct realizations of the same transfer functions are possible, corresponding to different canonical forms (controller, observer, etc. [KA1]). Cascade realizations have been used extensively in network synthesis and related areas [VV]. Perusal of the vast literature on adaptive systems reveals t an overwhelming majority of the work in this area is based on the direct lization. A notable exception is a special case of cascade realizations: the der form. A typical example of a ladder form for an autoregressive (AR) or pole model is depicted in Figure 3.2.

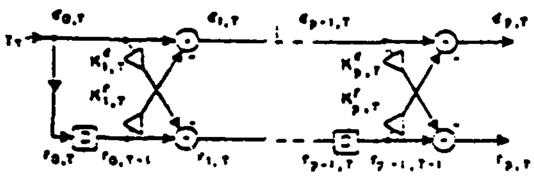


Figure 3.2. An Autoregressive Ladder Form

der type structures originated in Network and Scattering Theory and ame popular more recently in speech processing [MG] and actually earlier in mic applications. However, their overall impact on the area of adaptive pro- sing is still relatively small when compared to the popularity of direct reali- ons.

Ladder structures have many attractive features which make them prefer- to direct realizations. One such feature is the orthogonality (decoupling) perty: the signals propagating in a ladder form whitening filter (the forward backward innovations) are uncorrelated and therefore the coefficients of filter can be adjusted independently. One manifestation of this property is fact that when the order of the ladder filter is increased, one can add addi- al sections to the filter *without changing* the rest of the filter. In other is, the (p+1)-th order filter which fits a given data set $\{y(t)\}_{t=0}^{T}$, is the e as the p-th order filter, except for the last section, i.e., the lower order els are *nested* in the higher order models. Therefore, ladder filters make it ible to get *simultaneous estimates of models of different orders* (up to a maximum filter order), with no extra computation! This flexibility is extremely useful in tackling the problem of model order determination, which is one of the most difficult aspects of signal modeling. It should be noted that this nesting property does not hold for direct realizations.

The cascaded structure of the ladder filter (consisting of identical sections) is very convenient from an implementation standpoint, especially if special purpose hardware (e.g., VLSI) is used. Other properties of ladder forms which have been reported in speech processing literature are their excellent numerical behavior, robustness and insensitivity to round-off noise. Ladder forms have also been used very successfully for adaptive filtering applications like noise cancelling, and channel equalization.

The reasons for the many favorable properties of ladder forms have not been fully explored. One important issue is the interpretation of the reflection coefficients as partial correlations between the forward and backward innovations. These quantities have a more direct physical meaning than the ARMA coefficients $\{a_i, b_i\}$, which are related in a complicated way to the poles and zeroes of the system (through finding roots of a polynomial). Furthermore, the mapping from ARMA coefficients to poles and zeroes is very sensitive, i.e., small changes in the coefficients can lead to relatively large changes in pole-zero locations. The sensitivity to changes in reflection coefficients seems to be considerably smaller. Ladder models are also naturally related to physical models like transmission lines or a layered scattering medium. Such models correspond, for example, to sound waves propagating in shallow water. The reflection coefficients correspond to actual reflections of waves propagating in the transmission line. The $\{a_i, b_i\}$ coefficients do not seem to have a direct physical interpretation of this kind.

In view of these observations it may seem somewhat surprising that ladder

3.3 Event Detection

The basic concept of event detection and its usefulness for DSN application was discussed already in [DSN]. Since then, we performed a more detailed study of the ladder form as an event detector, in the context of speech processing. The following is a summary of this work, as described in [LM2].

The many nice features and advantages, such as a fast parameter tracking and excellent convergence, of a class of recursive least-squares ladder estimation algorithms have been reported by us in [MLNV], [MVL] and [ML2]. The application of these results to speech modeling and synthesis were reported in [ML1]. In [LM2] we present a novel innovations based pitch detection technique using the ladder algorithms. The important features of this pitch detector include:

(i) on-line time-domain operation directly on the speech waveforms;

(ii) all the necessary variables for the pitch detector are already computed by the modeling ladder algorithms and therefore is well-suited for an integrated implementation, (e.g., VLSI).

The basic assumption is that the speech driving process consists of a approximately Gaussian part (unvoiced), and a jump component (voiced). The pitch positions located by processing the innovations alone are not very accurate because the non-whiteness of the Gaussian component will act to cloud the position of the pulses, mainly due to phase-distortions. In our ladder algorithms, a gain parameter called γ_p, is computed recursively in the time-update recursions. This γ_p parameter turns out to be a very useful Gaussian log-likelihood type function for separating the mixed driving process into Gaussian and non-Gaussian components.

Briefly, the γ_p variable appears in the joint Gaussian distribution of speech samples $\{y_T, y_{T-1}, \ldots, y_{T-p}\}$ as ılopment of a particular class of filter configurations, the ladder or lattice ıs, which allows designs to meet the stringent accuracy and speed require- ıts of complex applications like speech synthesis and analysis. Such struc- ıs go beyond the realm of filtering and induce novel pipelined and parallel ırithms for the solution of important problems in matrix arithmetic.

In this paper, we first describe the variety of operations performed by an ptive filter for speech analysis. A short review of the CORDIC algorithms ws that they are ideally suited to realize these operations. To complete the ly of the filter implementation, a real-time speech analysis chip is discussed se only processing elements are CORDIC blocks.

Next we turn our attention to more general numerical problems involving rices. These problems are classified in terms of the order of the matrices. eed large size problems generally possess some structure, for instance rseness, so that they can be described by fewer parameters than if they had arbitrary number of elements. Methods for the solution of large problems dictated by their structure, which must be preserved during the computa- ıs so that both storage and execution time stay low. After the description of hly parallel architectures based on CORDIC blocks for small problems, i.e., ıout any structural assumption, we survey several architectures for large blems. Image reconstruction, beamforming for large antenna arrays (with 0 or more sensors), seismic data processing, boundary-value problems in tial differential equations (such as the ones encountered in weather forecast-), are just a few examples of large problems that can be efficiently solved with se architectures.

of signal processing techniques carries over to numerical linear algebra: direct techniques are related to triangular matrix factorization and inversion, while transform techniques are closer to orthogonalization and eigenvalue determination.

We address the issue of efficiently computing elementary operations which are prominent in signal processing and matrix computations. Most signal processing structures, whether parallel or not, tend to emphasize the need for fast multiplication (e.g., [4]). However, we show that fast vector rotation and related coordinate transformations are much more fundamental elementary operations for these problems. Algorithms that perform these operations were introduced by J.E. Volder [5], who named them CORDIC for COordinate Rotation DIgital Computer. Thus we advocate the use of CORDIC-like blocks as elementary processors.

Parallel execution is needed to achieve large throughputs; "highly concurrent" systems, consisting of many processors operating simultaneously, are developed for this purpose. Computer scientists have designed multiprocessor systems and array processors in the past, see e.g. [3b]-[3d]. With the advent of VLSI, such collections of processors become economically feasible, since many processing elements may be realized on a single chip. This has lead to much renewed interest in parallel processing as witness [1]-[3]. Up to now, the effort has been mainly to restructure for parallel execution existing algorithms, designed for execution on single processor machines. However, better performance can be expected from algorithms designed to exploit the computational capabilities of parallel processors.

Much can be learned from the signal processing community, which, over the last two decades, has paid considerable attention to issues related to the digital implementation of filters or transforms. One outcome of this effort has been the resulted in a push for the development of faster computing structures as well as algorithms of lower computational complexity. For particular problems, special purpose hardware has also been developed, but, owing to its lack of generality, this approach will not be considered here.

General purpose uniprocessor computers, especially microcomputers, have been utilized in the high speed signal processing arena with only limited success for primarily three reasons. First, they cannot in general compute a variety of elementary operations such as multiplication, vector rotation and trigonometric functions efficiently. These operations are very common in signal processing algorithms. Secondly, general purpose computer architectures provide only cumbersome address arithmetic for data structures, such as circular buffers, that occur frequently in communications applications. Finally, signal processing algorithms exhibit a substantial amount of parallelism that is not efficiently exploited in a uniprocessor system. A notable exception is the AMD2900 family which allows some parallelism through the extensive use of two port random access memories (RAM's).

Signal processing techniques roughly fall into two categories:

- direct or recursive techniques, where the signal simply passes through a digital filter, and
- transform techniques, where the signal is batch-processed. A global transformation is first applied to a batch of data, then simple operations like windowing are performed in the transform domain, and finally, the resulting signal is transformed back into the original domain.

Matrix computations share many features with digital signal processing. They call for the same set of elementary operations, essentially scaling, rotations, trigonometric functions and square-roots. Furthermore, and this illustrates the extent of the connection between these two areas, the above partition more, the duality between Generalized Levinson/Fast Cholesky algorithms is clearly displayed as a construction/extraction of elementary sections of the same transfer matrix T.

(iv) A VLSI Speech Analysis Chip Set Based on the Square-Root Normalized Ladder Form [AMLA]

One of the most attractive features of ladder estimation algorithms is their highly modular structure and suitability for VLSI (Very Large Scale Integration) implementations. To demonstrate this fact, we presented in [AMLA] and [AAM] a chip set design based on our square-root normalized ladder form. It is shown that the equations are amenable to implementation using the so-called CORDIC (Coordinate Rotation Digital Computer) algorithms because the time and order updates are easily representable as orthogonal transformations or rotations. An integrated implementation which exploits the concurrency of the ladder recursions together with possible hardware, speed and area tradeoffs are presented. The general applicability of the chip set to other signal processing tasks is demonstrated by showing that the discrete Fourier transform (DFT) is naturally suited to the architecture. We note that our theoretical development of the rotation based theory for ladder forms [MML] [MMLD] was actually motivated by implementation issues using CORDIC'S.

(v) Highly Concurrent Computing Structures for Digital Signal Processing and Matrix Arithmetic [ADM]

One important measure of the utility of digital signal processing algorithms has traditionally been computational complexity. The digital implementation of such algorithms has been frequently difficult (or impossible) since they tend to be compute bound. Consequently, the quest for real time signal processing has

(ii) Hilbert Space Array Methods for Ladder Realizations [MML]

In this paper we presented a Hilbert space array approach for deriving fast estimation algorithms and adaptive signal processing algorithms, that are recursive in time and order. Ladder (or lattice) forms turn out to be the natural realizations of these algorithms. From a stochastic point of view, the natural class of processes associated with these techniques include stationary but also nonstationary processes. They are encountered in adaptive signal processing, speech modeling and radar and sonar, high-resolution spectral estimation, distance measures, etc. The use of projections and orthonormalization, e.g., via Gram-Schmidt procedures induces real and complex rotation as basis operations, resulting in magnitude normalized variables and numerically stable computations.

(iii) Σ-Contractive Embeddings of the Discrete Lyapunov Equation [DGMV]

In the paper [DM1], a Levinson-type algorithm for fast inversion of covariance matrices with low displacement rank has been derived. In [DGMV] we first present a normalized version of these recursions (the multichannel case is considered throughout the paper). Then, the quantities involved in these recursions are embedded into a state-space description. This realization exhibits an orthogonality property that translates into the para-unitarity and contractiveness of its associated transfer matrix $T(z)$.

In general, a para-unitary transfer matrix can be shown to admit a cascade realization with sections of degree one, i.e., with one memory element. We demonstrate that the Generalized Levinson algorithm builds recursively a cascade realization of $T(z)$, where the elementary sections have p memory elements, p being the number of channels. The General Fast Cholesky algorithm by columns [M1] is also shown to yield the same cascade realization. Further-certain Riccati-type equations. Indeed, it turns out that the state-space system matrix of the ladder form is identical to a certain canonical matrix for the discrete-time Lyapunov equation called Schwarz form, investigated for instance by Mansour [MAN], in the context of Lyapunov stability test. This connection between the ladder form and the Schwarz form was noted by Morf [M2], and was elaborated on by Lee [LEE].

Since Lyapunov theory is intimately related to well known classical results of the polynomial approach, such as Routh-Hurwitz, Hermite, and Schur-Cohn, etc. the ladder form solution to Lyapunov and Riccati-type problems often offers much simpler and elucidating proofs to many classical results. As an example, the solution to the classical Schur-Cohn problem can be established via the ladder form, demonstrating the simplicity of the ladder canonical form method, compared to other approaches such as the controller form solution [BW] and Hankel matrix approach [DAT].

We discuss other implications of the state-space structures of the ladder form such as demonstrating the extremely efficient way of generating stationary states of any white noise driven system in the minimal number of steps (equal to the order of the system) and simultaneously check for stability or stationarity of a process.

We may note that representation of the state-space by other orthogonal polynomials have been considered in the literature. For example, Good [GO] has considered the Chebyshev polynomials in the so-called colleague matrix form, Barnett [BAR], and Anderson [AND] have considered extensions to generalized orthogonal polynomials in the so-called comrade matrix forms.

We briefly describe the transformations between ladder and controller canonical forms and outline the derivations of some of the properties mentioned above; details will appear in a forthcoming paper.

Theoretical Studies

In addition to the derivation of ladder algorithms of different types, a considerable amount of work was done on various theoretical issues related to ladder structures. As mentioned earlier, ladder forms play an important role in system theory. In fact they seem to provide a particularly convenient "coordinate system" for studying a large class of system theoretic problems. We briefly summarize here some of the main issues that were addressed so far.

(i) State Space Structure of Ladder Canonical Forms [ML3]

The purpose of this paper is to present the structure of the ladder canonical forms in a state-space model context and to recast and review various results involving ladder forms from a state-space model view, in contrast to the input/output (or transfer function) point of view stressed in the past.

The transformations between ladder- and other state- space canonical forms are presented. By means of state-space models we established some of the more interesting properties of the ladder forms, such as being the natural canonical form for the discrete-time Lyapunov and the matrix Riccati equation for stationary and so-called nearly stationary or finite rank processes.

Conventional canonical state-space realizations are obtained by constructing bases of either the Hankel matrix of the Markov parameters or of the controllability (or observability) matrices of the system. The ladder form realizations are obtained by an orthonormalization (using a Gram-Schmidt type procedure) of the state-space with the Szegö orthogonal polynomials as basis [M1],[M2], or equivalently via (numerically stable) orthogonal transformations. As a result of this orthonormalization, the state covariance matrix has the distinct advantage of being diagonal (or unity in certain normalized case) when the input is white, thus making it an attractive choice for solution to Lyapunov and

The covariance ladder form has numerous applications in adaptive processing and it also provides an efficient solution technique to numerous problems in system theory, such as:

(i) Minimal realization of multi-input and multi-output impulse response sequences, and of covariance sequences (the so-called stochastic realization problem);

(ii) Transforming right Matrix Fraction Descriptions (MFD) to left MFD's and vice versa. This problem arises in the ARMA modeling approach to the source location problem, as described in Section 2.4;

(iii) Model approximation, by choosing a desired number of sections in a ladder realization;

(iv) Stability tests for continuous and discrete time-invariant systems.

Convergence Analysis [EM]

An important step in the development of any new recursive algorithm is the analysis of its convergence. In the case of the AR ladder form such an analysis is not really needed, since the algorithm provides an exact solution of a least-squares problem, and the asymptotic properties of recursive least-squares estimators are by now well known. The situation is very different in the case of the ARMA ladder which involves a nonlinear "bootstrapping" technique. In [EM] a preliminary convergence analysis is presented. This paper gives an asymptotic analysis of a particular ladder algorithm, designed for auto-regressive moving-average (ARMA) modeling of time-series. A thorough analysis of a second order case reveals that the asymptotic properties can be improved by modifying the algorithm. The modified algorithm is then shown to be asymptotically equivalent to the well-known extended least squares method in the general case.

This would be the case, for example, if u_T is generated by linear feedback in a control system. The vector process $z_T = [y_T', u_T']'$ will then have an AR representation

$$\underbrace{\begin{bmatrix} \varepsilon_T^y \\ \varepsilon_T^u \end{bmatrix}}_{\varepsilon_T} = \begin{bmatrix} y_T \\ u_T \end{bmatrix} - \begin{bmatrix} \hat{y}_{T|T-1} \\ \hat{u}_{T|T-1} \end{bmatrix} = \underbrace{\begin{bmatrix} y_T \\ u_T \end{bmatrix}}_{z_T} + \sum_{i=1}^{n} \underbrace{\begin{bmatrix} a_i & -b_i \\ f_i & -g_i \end{bmatrix}}_{A_i} \underbrace{\begin{bmatrix} y_{T-i} \\ u_{T-i} \end{bmatrix}}_{z_{T-i}} \tag{3.6}$$

By applying any AR modeling technique to the time-series $\{z_i\}_{j=1}^{T}$, we can obtain estimates of the ARMA parameters a_i, b_i. As a by-product we also get the parameters c_i, d_i of the model for u_T. If these are not needed, it is possible to simplify the algorithm by eliminating the equations involving f_i, g_i.

When the input u_T is not known, such as in the case of modeling an ARMA times series, the situation is slightly more complicated. The unknown input need to be replaced by its estimate, which under certain assumption can be shown to be ε_T^u. For a detailed presentation of this case see [FR2], [LEE].

Similar ideas can be used to develop ladder forms for joint process estimation [LM1], [LEE] and for ARMAX models [FR3].

The square-root normalized ladder algorithms discussed so far provide a solution to the so-called pre-windowed form of the normal equations. Another version of the normal equations that is commonly used in speech processing and other applications is the covariance form. In [PFM] we developed the growing memory and sliding memory covariance ladder algorithms, using a simplified derivation method. Reference [PFM] also includes some new ladder form realizations of the identified models, leading to convenient methods for computing the parameters from estimated reflection coefficients. Another topic presented there is a complete solution to the problem of possible singularities in the ladder update equations.

greater simplicity of the normalized recursions leads us to prefer them over their unnormalized counterparts. Sometimes it is desired to compute the unnormalized innovations for comparisons with other spectral estimation techniques. This can be done by proper scaling of the normalized variables:

$$\varepsilon_{p,T} = (1 - \gamma_{p-1,T-1})^{1/2} R_{p,T}^{\varepsilon/2}\ \varepsilon_{p,T} = \text{unnormalized innovations} \tag{3.3a}$$

$$\varepsilon_{p,T} = (1 - \gamma_{p-1,T-1})^{1/2} \varepsilon_{p,T} = \text{variance normalized innovations} \tag{3.3b}$$

where

$$R_{p,T}^{\varepsilon/2} = R_T^{1/2} \prod_{i=1}^{p} (1 - K_{i,T} K_{i,T}')^{1/2} = \text{forward innovations covariance} \tag{3.4a}$$

$$(1 - \gamma_{p-1,T-1})^{1/2} = \prod_{i=1}^{p} (1 - r_{i,T-1}' r_{i,T-1})^{1/2} \tag{3.4b}$$

Equations (3.2a)-(3.2c) represent an adaptive whitening filter. Given a (vector) data sequence y_T, they generate an innovations sequence ε_T as well as a set of reflection coefficients $\{K_{p,T}\}$ which fully characterize the filter.

Some Extensions

So far we discussed only ladder forms for multichannel AR processes. As mentioned before, by using certain embedding techniques, it is possible to extend these results to more general situations. In [LEE], [LFM] and [FR2] we presented such an extension to ARMA models. As an illustration of the embedding idea, we present here a brief description of its application to the ARMA case.

Consider the problem of estimating the parameters of the ARMA model given by Eq. (3.1) from input/output data $\{y_i, u_i\}_{i=1}^{T}$. Let us assume that the input process u_T is in itself an ARMA process:

$$u_T = \sum_{i=1}^{n} f_i\, y_{T-i} - \sum_{i=1}^{n} g_i\, u_{T-i}\ . \tag{3.5}$$

Thus, with just one extra equation we can solve the model fitting problem!

Several versions of the exact least squares ladder forms were developed: the unnormalized, the variance normalized and the magnitude normalized ladder forms. The normalizations refer to the signals propagating in the ladder filter, i.e., the forwards and backwards innovations. The variance normalized form involves signals with unit variance and the magnitude normalized form has signals with magnitude less than one. The simplest of these three forms is the magnitude normalized algorithm which is summaried below for the all-pole (AR) case. It should be noted that this normalization makes it possible to compute the ladder recursions with fixed-point arithmetic, which is ideally suited for micro-processor and very large scale integration (VLSI) implementation!

The basic square-root normalized AR ladder recursions are given by

$$K_{p+1,T} = [I - \varepsilon_{p,T}\, \varepsilon'_{p,T}]^{1/2} K_{p+1,T-1} [I - r_{p,T-1}\, r'_{p,T-1}]^{T/2} + \varepsilon_{p,T}\, r'_{p,T-1} \tag{3.2a}$$

$$\varepsilon_{p+1,T} = [I - K_{p+1,T}\, K'_{p+1,T}]^{-1/2} [\varepsilon_{p,T} - K_{p+1,T}\, r_{p,T-1}]\, (1 - r_{p,T-1}\, r'_{p,T-1})^{-1/2} \tag{3.2b}$$

$$r_{p+1,T} = [I - K'_{p+1,T}\, K_{p+1,T}]^{-1/2} [r_{p,T-1} - K'_{p+1,T}\, \varepsilon_{p,T}]\, (1 - \varepsilon'_{p,T}\, \varepsilon_{p,T})^{-1/2} \tag{3.2c}$$

where

K_p = reflection coefficient matrix

ε_p = magnitude normalized forward innovations

r_p = magnitude normalized backward innovations

The notation $[\,\cdot\,]^{1/2}$ denotes the matrix square root. The derivation of the all-pole normalized ladder forms can be found in [LEE], [LM1]. Alternatively, one may use the unnormalized ladder forms [ML1] [ML2]. However, preliminary tests seem to indicate that the normalized recursions perform at least as well as (and maybe better than) the unnormalized version. This fact combined with the windowing (e.g., as in the *covariance* or *auto-correlation method*). An assumption of stationarity (Toeplitz covariance matrix) is used in these computations.

Block processing is often useful, but in adaptive processing applications where computations have to be carried out in real-time, recursive methods are needed. Recursive methods have distinct advantages even in situations where block processing could be used. They are generally easier to implement and lead to greater flexibility. In signal processing applications where the signals have time-varying nonstationary statistics it is useful to be able to update continuously the estimates of the model parameters (e.g., the reflection coefficients of the ladder form), rather than do so at fixed time intervals. This allows faster adaptation of the filter to variations in the data. Because of these and other reasons there is great interest in developing recursive algorithms for fitting ladder filters to an observed time series. Some results have been reported, using gradient search techniques which adaptively vary the reflection coefficients so as to minimize a square error criterion. These gradient techniques often show slow convergence which limits their usefulness.

The exact least-squares method is a recursive technique for computing the ladder coefficients. It computes at each time step, a set of reflection coefficients which minimizes the sum of the squared errors for the data sequences up that time. Thus, it provides an *exact solution* to the least squares model fitting problem! The computation is recursive both in time and in model order, which adds an extra degree of flexibility. What is rather remarkable is that this exact recursive solution is obtained at almost no increase in the computational requirements. The normalized ladder forms involve only three update equations per time step and per section of the filter. Two of these equations are needed to implement the ladder filter (i.e., propagate the forwards the backwards innovations), while the third updates the reflection coefficients.

forms are not more widely used in adaptive processing. One reason for this may be the fact that until fairly recently techniques for fitting ladder structures to observed data were restricted to very simple classes of systems of the single-input single-output all-pole (or all-zero) type. Many adaptive processing problems involve ARMA models and multiple inputs and outputs. Using a certain embedding approach we were able to extend considerably the class of problems to which ladder structures can be applied.

A second reason which may have limited the use or ladder structures was the lack of efficient recursive computational methods for estimating ladder parameters (reflection coefficients). Until fairly recently, the only techniques available for ladder modeling were of the block processing type, i.e., they operated on data collected over a finite time interval. Adaptive processing requires frequent updating of the estimated parameters which is best achieved by a recursive algorithm. It is possible to use block processing algorithms in a sliding-window mode to obtain point-by-point updates, but this often requires prohibitive amounts of computation. The development of the exact least squares recursions for ladder forms provides a strikingly simple and efficient algorithm, which makes the use of ladder forms in adaptive processing much more attractive.

Recursive Ladder Forms

Various algorithms have been proposed for finding a ladder form representation of time series. These algorithms are mostly of the block-processing type. Typically, the algorithm is based on the fact that given the data covariance matrix, the reflection coefficients can be computed by a Levinson-type recursion. Since the data covariance is usually unknown, it has to be replaced by an estimate. This is done by using the sample covariance of the data with a proper

$$p(y_T, \dots, y_{T-p}) = |2\pi \mathbf{R}_p|^{-1/2} \exp\{-\frac{1}{2}\gamma_{p,T}\} . \qquad (3.7)$$

where

$$\gamma_{p,T} = [y_T, \dots, y_{T-p}] \mathbf{R}_{p,T}^{-1} [y_T, \dots, y_{T-p}]^T \qquad (3.8)$$

and $\mathbf{R}_p$ is the covariance matrix. Taking the logarithm, we have a log-likelihood function of the form

$$\lambda_p = \ln |\mathbf{R}_p| + \|\mathbf{y}\|^2_{R_p^{-1}}$$

$$= \ln |\mathbf{R}_p| + \gamma_p . \qquad (3.9)$$

It turns out that the increase in λ_p per sample, i.e., the derivative, $\delta \lambda_p$ of λ_p , is a very sensitive measure of the "unexpectedness" of a speech sample, i.e., a measure of the deviation of the actual distribution from the Gaussian hypothesis. This is actually a very useful approach in the so-called innovations based detection of outliers in the statistics literature, see [ANS], [AN].

Thus a very simple but powerful pitch pulse detector can be designed as follows:

(i) use a local maximum algorithm either on $\delta \lambda_p$, or $\delta \gamma_p$, returning a zero value if the maximum is below some threshold;

(ii) multiply the resulting log-likelihood by the innovations, ε_p, and then use a simple exponential threshold detector similar to that of [GR] to locate the pitch pulses.

A sample of the pitch detection results is shown in the sequence of figures below.

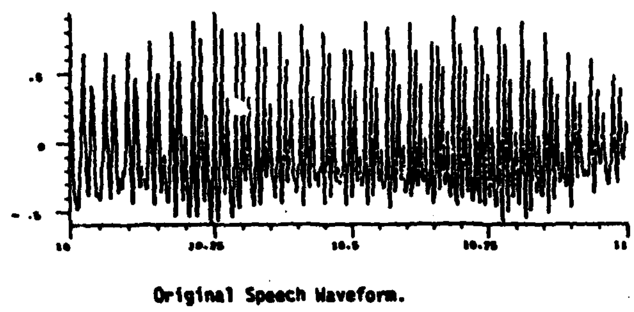


Figure 3.3. Original Speech Waveform

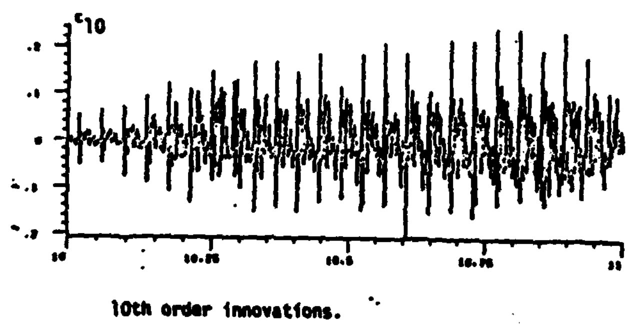


Figure 3.4. 10th Order Innovations

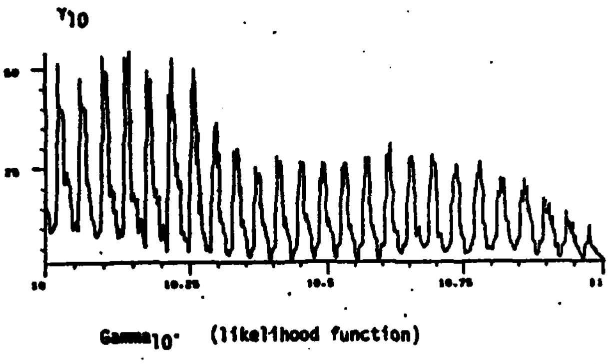


Figure 3.5. γ_{10} **(Likelihood Function)**

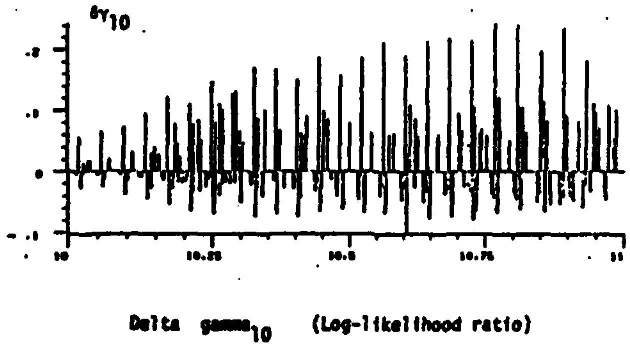


Figure 3.6. $\delta\,\gamma_{10}$ **(Log-Likelihood Ratio)**

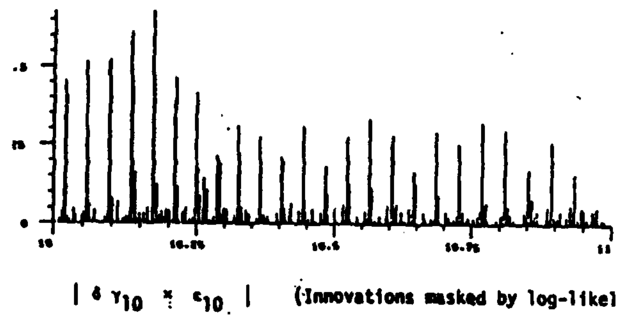


Figure 3.7. $|\delta\ \gamma_{10} \times \varepsilon_{10}|$ **(Innovations Masked by Log-Likelihood Ratio)**

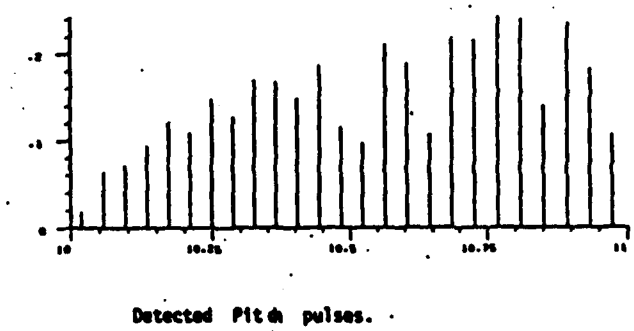


Figure 3.8. Detected Pitch Pulses

We may note that this particular pitch detection scheme was designed to illustrate the basic principles and the power of our ladder form algorithms, and is by no means an optimized solution to the problem of pitch detection. This is clear from the fact that plosive sounds also generate large γ's; however they tend to be isolated and not to occur in periodic intervals.

From a stochastic modeling point of view, one should be able to decompose the driving process into essentially continuous and discontinuous or jump type components. We view the above scheme as a first step in this direction.

The algorithm discussed above can be used as a sensitive detector of various events in a DSN system such as: the appearance (or disappearance) of a new source in the area under surveillance; a change of course, in the flight path of an airplane or other source of acoustic energy; a change in the spectral characteristics of the source. This event detector can be used to identify which portions of the large amount of data that are expected to be available in a DSN system, are of interest. We then can proceed to process in detail only the selected data, thus achieving considerable savings in computational requirements.

We note that other types popular log-likelihood functions distance or distortion measures can be computed efficiently via ladder forms, for a discussion of such results see [LM4].

Application to High Speed Modem and Packet Radar/Sonar

Distributed Sensor Networks clearly involve not only collecting and processings data, but communication/exchange of digital information as well. We also stated earlier, that event detection was an important component of data processing. Events, by definition, are discrete or discontinuous phenomena that generally require different sets of mathematical tools and processing algorithms than continuous or "slowly" varying objects, such as spectral parameters or track information of slowly moving sources. Digital information, is discrete by definition, hence a digital communication system has to be based on signals that exhibit discontinuous components or events.

Our work on tools for event detection, e.g., using ladder forms for computing likelihood functions, is now clearly applicable to processing digital communication data. Although such work has already been done in the area of digital communication in general and modems (modulators - demodulators) in particular, relatively few results are available on the computational complexity and implementation of optimal modem (e.g., 9.6 kbits for dial-up lines) designs. We only note that currently available modems are either far from the telephone channel capacity (e.g., 30 kbits for a single commercial dial-up telephone channel) or very expensive. The availability of high capacity digital links is, however, a crucial parameter in a high performance DSN.

Our work on fast computation of likelihood functions and other measures using ladder forms has now been extended to include the problem of optimal design of digital modems. As an example that would test our mathematical design tools, we decided to work out a design of binary FSK and PSK modems. This design is based on ladderforms, hence an economic implementation could be based on our LADMOS - VLSI chip design.

Our preliminary simulation studies have indicated that this modem has Bayes optimal detection characteristics in *colored* Gaussian noise.

In a DSN context, the *combination* of passive or active sensors (e.g., sonar/radar/acoustic), and analog or digital communication is an extremely promising solution, in our opinion, to the design constraints imposed on a DSN by its distributed character and most likely limited communication capacity over hard channels (cables, fiber optic, etc.).

We envision that cooperative sensors and possibly active transmitters for both source tracking and communication will lead to new types of solutions for DSN problems. For example, the bistatic radar mode of operation is very natural between communicating sensor nodes, if transmitters are linked to the sensors; hence cooperative tracking of sources, as well as "space-hopping" (in addition to frequency and time hopping radar/sonar modes) is clearly achievable. In addition if transmitting signals are modulated with digital data, *high bandwidth* digital communication channels become available between sensor nodes! I.e., we can envision *packet radar/sonar networks*. In order to achieve this goal we clearly need combined event detection and digital modem related signal processing tools of the type we are developing.

3.4 Doubling and Tree Algorithms [M3], [MD1], [ADM]

The inversion of Toeplitz covariance matrices arises in practically all estimation problems related to stationary time series. If $\{y_t\}$ is a stationary random process, and x is some statistically related random variable, then the least squares estimate of x based on observations $\{y_t\}_{t=0}^{T}$ is given by

$$\hat{x} = R_{xy} R_y^{-1} Y \ , \tag{3.10}$$

where R_y is the covariance matrix of the data vector $Y = [y_0', \ldots, y_T']'$ and R_{yx} is the cross covariance, i.e., $R_{yx} = E\{yx\}$. Because of the assumption of stationarity, R_y has a Toeplitz structure. The random variables $\{y_t\}_{t=0}^{T}$ do not necessarily represent a time sequence. They may be, for example, measurements at the outputs of multiple sensors, in which case the index t represents the sensor number (i.e., it is a spatial rather than a temporal index). This type of problem arises in adaptive beamforming, adaptive antenna arrays, and in a DSN system consisting of multiple acoustic or electromagnetic sensors. Development of efficient solution techniques for Toeplitz sets of equations is therefore of great interest in the context of DSN signal processing.

Using the HGCD algorithm of Aho, Hopcroft, Ullman [AHU], Gustavson and Yun [GY] constructed an algorithm to invert a Toeplitz matrix in $O(n \log^2 n)$. The main principle used in the HGCD (half-greatest-common-divisor) as well as in our new algorithm is the "divide and conquer" or "doubling" approach. In both cases an implicit form of the inverse of an $n \times n$ Toeplitz matrix T is found in $O(n \log^2 n)$ operations; however, our algorithms can work with a larger class of matrices, and they have the potential for being more efficient, e.g., $O(n \log n)$.

We are interested in using the idea of doubling directly on Toeplitz matrices without referring to the HGCD algorithm and to find expressions for inverting a more general class of matrices, that is related to Toeplitz matrices. The objective was to reduce the $O(n^2)$ operations of the well-known algorithm of Levinson [LEV] and its α-Toeplitz matrix extensions [M1], [FMKL], [KKM].

First we recall the definition and properties of displacement ranks required in the proof of the basic algorithm. Then we introduce the Fast Toeplitz Matrix Inversion Algorithm. It is based on partitioning of a matrix

$$M_{2n} = \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} . \tag{3.11}$$

where the indices indicate the dimensions of matrices, and its inverse

$$M_{2n}^{-1} = \begin{bmatrix} \tilde{A}_n & \tilde{B}_n \\ C_n & D_n \end{bmatrix} . \tag{3.12}$$

The Schur complements of the submatrices A_n and D_n are defined as

$$\bar{A}_n = A_n - B_n D_n^{-1} C_n , \quad \bar{D}_n = D_n - C_n A_n^{-1} B_n . \tag{3.13}$$

Now we have the following well known relations

$$\tilde{A}_n = \bar{A}_n^{-1} = A_n^{-1} + A_n^{-1} B_n \tilde{D}_n C_n A_n^{-1} \tag{3.14}$$

$$\tilde{B}_n = -A_n^{-1} B_n \tilde{D}_n = -\tilde{A}_n B_n D_n^{-1} \tag{3.15}$$

$$\tilde{C}_n = -\tilde{D}_n C_n A_n^{-1} = -D_n^{-1} C_n \tilde{A}_n \tag{3.16}$$

$$\tilde{D} = \bar{D}_n^{-1} = D_n^{-1} + D_n^{-1} C_n \tilde{A}_n B_n D_n^{-1} \tag{3.17}$$

The algorithm HTI (half α-Toeplitz matrix inversion) follows by applying the properties of Toeplitz and sums of products of Toeplitz matrices to these formulas, and calling the HTI algorithm recursively for the necessary inversions of submatrices. The crucial observation is that the Schur complements have an efficient representation that is just as complex as the submatrices themselves, i.e., they have the same so-called displacement rank, a number that measures the distance from a Toeplitz matrix, hence the same computational complexity for multiplications and inversions.

We conclude by mentioning some applications of these results. Matrix inversion algorithms that are based on partitioning lead to the problem of inverting the Schur complement of a submatrix. If one is interested in algorithms that can take advantage of the structure of the matrix to be inverted, the question arises, what properties are *invariant* under the Schur complement action. As an illustration, we sketch a partial list of matrices that can be shown to have such invariance;

- Hankel and Hankel plus Toeplitz type equations;
- Banded and rational (ratio of banded) type matrices, $O(n \log b)$;
- Block Toeplitz circulant or Hankel matrices;
- Matrices that are related to two-dimensional (2-D) and multi-dimensional (M-D) problems, e.g., Toeplitz block Toeplitz matrices;
- Other combinations of the above.

There are various other interesting topics that are related to doubling algorithms of this type, e.g.,

- Stochastic interpretation of doubling/halving algorithms [DM1], [MD1];
- Fractal ladder form realization of matrix inversion.
- open problems, especially computational refinements,
- other applications, estimation, detection, pattern recognition,
- image reconstruction from projection, etc.

The study of structures of sets that are inherited in subset is a topic by itself. Generally this leads to recursive function theory. A very interesting class of objects, perhaps up to now more of a curiosity, consists of the so-called fractals. They are best thought of as objects created by recursive function calls or by "mirror galleries". For an example see the "fractal ladder realization" in

4].

Many, especially yet undiscovered applications can be obtained by suitable erations that convert structure of matrices into sparseness. Simple examples this type are matrices that have higher order displacement rank, e.g., [KKM]. ıch concepts are useful for matrices with n^{th}-order polynomial variation along agonals and off-diagonals, compared to constants for Toeplitz matrices.

3.5 Adaptive Signal Processing

The distributed sensor network operates in an uncertain environment. Various system parameters are unknown to the processing nodes either because of random effects or because of their time varying nature. Therefore, fixed signal processing schemes may be inadequate in certain DSN applications, and an adaptive processing technique will have to be used. Several adaptive techniques were developed as part of our research program, and are briefly summarized below.

Adaptive Linear-Phase Filters [FM2]

An important class of filters commonly used in digital signal processing is the class of finite impulse response (FIR) linear-phase filters. Such filters are needed in applications where frequency dispersion due to nonlinear phase is harmful, e.g., speech processing and data transmission. Signals in the passband of linear-phase filters are reproduced exactly at the filter output, except for a delay corresponding to the slope of the phase vs. frequency plot. An extensive literature exists on the design of linear-phase filters with a desired frequency response and various efficient design algorithms have been developed. Most of this literature deals, however, with non-adaptive processing, i.e., designing the filter to have a known frequency response (or impulse response). Relatively little work seems to have been done on adaptive linear-phase filters, i.e., filters whose characteristics are determined on the basis of an observed time-series, and not on apriori specifications.

An FIR linear-phase filter is characterized by a symmetric impulse response. Let $A(z)$ represent the transfer function of an FIR filter, where

$$A(z) = a_0 + a_1 z^{-1} + \cdots + a_n z^{-n} \qquad (3.18)$$

$$z^{-1} = \text{unit delay operation, i.e., } z^{-1}x_t = x_{t-1} \ .$$

'his filter will be a linear-phase filter if $a_i = a_{n-i}$. In this paper we will s assume that $a_0 = 1$. Such filters appear in adaptive processing applica- often without being explicity identified as having linear phase. As an ple consider the high resolution spectral estimation of narrowband signals, toregressive (AR) modeling. Various spectral estimation techniques such e Maximum Entropy method [BUR], or Pisarenko's method [PIS], involve :ting of as AR predictor $A(z)$ to the observed data sequence y_t and then it to compute the spectrum

$$S_y(\omega) = \frac{1}{A(z)A(z^{-1})}\bigg|_{z=e^{j\omega}} \tag{3.19}$$

! the data y_t has a pure line spectrum (i.e., a sum of sinusoids of different encies) it is straightforward to show that $A(z)$ will be a symmetric polyno- with all its roots on the unit circle. In other words, the predictor $A(z)$ is cial case of a linear-phase filter.* In many of these techniques the sym- of $A(z)$ is imposed implicitly.

he interconnection between the predictor of a narrowband signal and -phase filtering should make the techniques described in this paper useful plications including adaptive line enhancement, adaptive noise cancelling laptive array processing. Another area where linear-phase filters are of l interest is adaptive channel equalization.

ıe objective of this paper is to formulate the adaptive linear-phase filter- oblem, discuss its interpretations in the context of linear least-squares tion and derive several types of adaptive algorithms. The emphasis is on neral features of the adaptive linear-phase filter and not on any specific

---

ıeral, if z_o is a zero of a linear-phase filter, so is $1/z_o$. For a zero on th unit cir- $= e^{j\omega_o}$, and therefore $1/z_o = e^{-j\omega_o}$. The complex pair $\{e^{j\omega_o}, e^{-j\omega_o}\}$ ts a sinusoid of frequency ω_o.

applications. While many aspects of these adaptive filters have been discussed in the literature, no unified treatment of this subject seems to be available.

Adaptive Line Enhancement [NM]

Recently there has been great interest in the problem of separating narrow band and periodic signals from wide band signals or noise, using least-squares predictors. A considerable amount of work was based on gradient type techniques, referred to as Adaptive Line Enhancer (ALE), by B. Widrow et al. [WG]. In [NM] we analyze the optimal behavior of the ALE, which coincides with the exact least-squares predictor in steady state. The ALE and the noise canceller can be viewed as a special case of the joint estimator process estimator or instrumental variable method, see e.g., [ML2] and [FR4], where the reference input is a delayed (or suitably pre-filtered) version of the primary input, as shown in Figure 3.9.

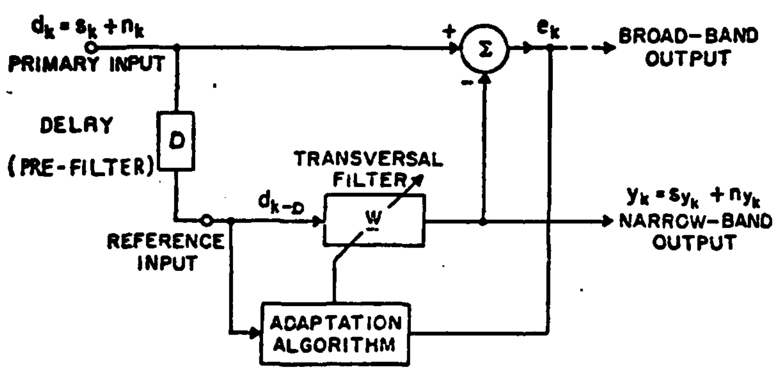


Figure 3.9. Block Diagram of the ALE

A more detailed examination led us to define several different but related ›tions of displacement rank, leading to a classification of algorithms and to the ıestion of minimality of the various associated representations.

In the discrete case, a new Toeplitz block matrix embedding principle prodes us not only with an elegant framework for relating various algorithms for ›lving Toeplitz systems, but also with an interpretation of the embedding ıatrix as a joint covariance matrix. Because of the larger size of this matrix, ıe number of random variables is roughly alpha times the original number of ıriables, hence we encounter an alpha multiplicity in a process representation.

This leads to the first physical insight that we can now provide: alpha is a ıultiplicity in the representation of a process, as the sum of the outputs of lpha linear time-invariant (but not necessarily finite-dimensional) systems tarting at time zero, i.e. the superposition of their transients. An alternative ıterpretation turned out to be to model the process as the prediction error of ne process given alpha other "pseudo" processes; pseudo processes because he estimation errors are by definition orthogonal to the "given" processes, but his implies that these "given" processes are not observable in the process to be ıodeled by the projection theorem. The alpha underlying innovations in the given" processes can now be viewed as "pseudo" innovations. Like the innovaions they are white processes that can generate the process causally; however, ınlike the true innovations they can in general not be obtained in a causal way, e. they are smoothing residuals, and they have in general an unobservable omponent. This component can be viewed as an additive "dither" noise or enciphering key" that allows the alpha pseudo innovations to be white. In paricular the last reference points out some potential applications of this epresentation.

A careful analysis shows that in general a mixed representation can be sumption of stationarity is very widely used. One reason for this dicho- ıppear to be observations of abrupt transition between the complexity of ıary and non-stationary results. We have recently introduced an index of ıarity (alpha), [FMKL] that has the potential of providing a more gradual uction of non- stationarities into the modeling of stochastic processes.

ıe significance of this index alpha is that the larger the value of alpha the ıon-stationary a process is. In addition, the complexity of models and cal- ›ns increases only proportional to this index, in a similar way as the order ndex of complexity for finite dimensional linear systems. However, there ·ocesses with finite alpha and infinite order (i.e., non-rational) and vice hence the two notions are quite independent, except for constant param- inite dimensional linear systems where alpha is bounded above by the [KA2], [MSK]. The difference is seen from the fact that finite order is lent with semi-separability of the covariance kernel, say $K(s,t)$, whereas is given by the rank of the separable displacement function $,t) = dK(s,t)/dt + dK(s,t)/ds$, see e.g., [KLM], where —| is an or of the divergence type. Livshits [LY] defined an equivalent infinitesimal ation function, and its separability rank the rank of the process. For te processes we defined in [M1] the "shift" or tensor rank for matrices o Toeplitz matrices, or what we call now the displacement rank - alpha. natrices are basically rational functions of Toeplitz matrices, whereas ›rder systems lead to matrices of rational functions.

any estimation, communication and control problems require the solution plitz and related systems of equations or integral equations of the dis- ıent type. Such solutions can be obtained efficiently via the use of exist- ıthods, such as the Krein-Levinson type algorithms [KLM], and the more doubling type algorithms [M3].

nels of Livsic, [L].

The displacement rank used so far is actually called a "strict" displacement rank in contrast with two other displacement ranks, the so-called "+" and "-" displacement ranks, see e.g. [KKM]. All these indices are closely related; their difference is bounded by two. It turns out that the Christoffel-Darboux type representation of the inverse obtained using the strict displacement rank [FMKL] is not always minimal. However it is possible, applying the semi-group property of Schur complementation, see [A], to find a minimal representation of the inverse that can be exploited to obtain still another Levinson-type algorithm for inverting matrices close to Toeplitz. The interest of this new algorithm lies more in the insight it gives on stochastic process representation and ladder forms than into the computational savings which are minor. However the same type of minimal representation can also be obtained for quantities of interest (e.g. Schur complement) in the doubling algorithm described in [M3] and the computational complexity is significantly reduced there. The quantities involved in the minimal Levinson-type algorithm also appear very closely related to the ones in the doubling algorithm.

The notion of displacement rank and the associated Toeplitz algebra are known to extend to operators, i.e., continuous time models, see e.g. [KLM]. Furthermore the Generalized Levinson equations for nondisplacement kernels were derived in that paper. In this last section a continuous time version of the doubling method [M3] will be described that yields yet another efficient computational procedure for determining Fredholm resolvents of nondisplacement kernels.

Pseudo Innovations Representations for Alpha-Stationary Processes [MD2]

Almost all processes found in practical situations are non-stationary, yet cks of different or varying sizes, a structure of special interest in state-space dels and minimal realizations.

Various embeddings of matrices close to Toeplitz in the sense of [M1], i.e., ıple rational functions of Toeplitz matrices, can be used. Two useful Toeplitz ck embeddings for symmetric matrices are inferred from their representa- n either as a Toeplitz matrix plus an algebraic sum of strictly lower triangular ›plitz matrices times their transpose or a Toeplitz matrix minus a sum of ‹er and upper triangular Toeplitz matrices times their transpose (mixed ›resentation). Application of the Toeplitz block version of the LWR algorithm the first embedding matrix provides a simple derivation of the Generalized rinson recursions of [M1], [FMKL] while the second embedding yields a rinson-type algorithm with improved numerical properties since the embed- ıg matrix is positive definite when the original matrix is positive definite.

Consider the case where the original matrix is the covariance matrix of a ıcrete-time stochastic process. The two Toeplitz block matrix embeddings esented above have (r+1)x(r+1) blocks where r is the displacement or shift nk of the covariance matrix, an index of non-stationarity of the process [M1]. reover the embedding matrix associated with the mixed representation can interpreted as a joint covariance matrix. A non-stationary process may erefore be modeled as the prediction error of one process given r other seudo" processes. The r underlying innovation processes are indeed "pseudo" ıovations; like the innovations they are white processes that can generate the ocess causally; however, unlike the true innovations they cannot in general be tained in a causal way,i.e., they are smoothing residuals. According to the xed representation, some of the r pseudo innovations are actually passed rough anti-causal filters instead of causal ones to form the pseudo processes. is last result appears closely related to the notions of direct and inverse chan-

- More recently developed doubling type algorithms [M3], that promise to be even more efficient, especially for large problems.

We present several results that allow us to see these seemingly different methods as part of a whole set of tools that are closely related. The new results can be viewed as missing links to a larger picture that is gradually emerging.

First we present a new Toeplitz block matrix embedding principle, that provides an elegant derivation of the Generalized Levinson algorithm, as well as connections to other problems, such as the inversion of Toeplitz plus Hankel matrices. The basic idea is rather simple; it is well known that the inverse of a block matrix can be expressed via the block inversion formulas as a rational combination of the component blocks, the idea is to reverse that process and view a given matrix as a rational combination of component blocks, where the blocks are components of a larger matrix. In this embedding process the hope is that the larger matrix displays a structure that is simpler than the original (smaller) matrix.

A historical example of this type is one of Levinson's attempts to solve Toeplitz systems in [W]. By partitioning a Toeplitz matrix into a two by two Toeplitz block matrix and applying a block reduction step, he winds up with a Toeplitz plus Hankel matrix inversion problem of half size. The problem was abandoned at this stage. Our point is that the inverse process can be used to embed the Toeplitz plus Hankel matrix inversion problem into a Toeplitz block matrix or, via an interleaving permutation, into a block Toeplitz matrix. Block Toeplitz matrices can be inverted with the multivariate Levinson-Whittle-Wiggins-Robinson (LWR) algorithm, while Toeplitz block matrices are inverted using the Toeplitz block version of the LWR algorithm, reminiscent of two-dimensional (2-D) generalizations of the LWR algorithm, see e.g. [LKM]. The Toeplitz block matrix form of the LWR algorithm can be extended to matrices with Toeplitz as auxiliary quantities. In estimation, these quantities are interpreted as covariances of residuals of one step predictors. In the doubling algorithm for α-Toeplitz matrices [M3], Schur complements of higher orders have to be considered. The tree associated with the algorithm is binary with, sitting at the nodes, matrices of order 1, 2, 4, ..., n obtained by combined partitioning and Schur complementation. The order 1 quantities are the same as the auxiliary quantities (residuals) associated with the leftmost (or rightmost) branch of the linear tree. The order 2^k quantities have the same interpretation taking order 2^k blocks as matrix elements. The algorithm is function recursive, a property that appears well adapted to the tree machines [BRO], [SEQ]. The α-Toeplitz matrix is inverted using a depth-first traversal of the tree; this total ordering implies a sequential overall computational scheme, although every (order) update may be computed in a parallel fashion (via FFT's). As for the linear tree, computations are further reduced if some quantities at the same level in the tree are known a priori to be identical or have low rank differences. The significance of such a structure is explored. A very simple example is given by the covariance matrix of a Brownian motion; then only one branch of the binary tree has to be considered. For possible VLSI architectures, see [ADM].

Mixed and Minimal Representations for Toeplitz and Related Systems [DM1]

Many signal processing, estimation and control problems require the solution of Toeplitz, Hankel or related systems of equations. Such solutions can be obtained efficiently via the use of several existing methods:

- The Generalized Levinson algorithm, see e.g. [FMKL], for matrices close to Toeplitz, such as sums of products of Toeplitz matrices.
- Ladder or Lattice Form realizations, see e.g. [MLNV], that result in stable and efficient implementations.

trices representative of each class. If the matrix is Toeplitz, all contiguous principal submatrices of the same order are identical so that only one branch of the tree has to be considered and the computations are equivalent to the Levinson algorithm [LEV]. Since order updates of (distinct) submatrices of the same dimension can be performed simultaneously, the algorithm is well-suited for parallel processing. More precisely, with a parallel processing implementation of the algorithm, any matrix is inverted with the same number of steps as a Toeplitz matrix of the same size.

In many important examples related to estimation and control, contiguous principal submatrices of the same order have a low rank difference. When the difference of the two contiguous principal submatrices of order $n - 1$ has (low) rank α, the rank of the difference of two contiguous principal submatrices of any order is bounded by α and the matrix has a shift low rank α [M1] and was referred to as α-Toeplitz in [FMKL]. The contiguous principal submatrices of an α-Toeplitz matrix are all distinct in general, hence, since all the branches of the tree would have to be explored, the algorithm developed in [DGK1] would not invert efficiently such a matrix. However, since two contiguous principal submatrices differ by a low rank correction, (a representation of) the inverse of one is easily deduced from the inverse of the other; this is a one step *time* update. Thus using both "vertical" (order) and "horizontal" (time) updates, only two principal submatrices of each order, i.e., two branches of the tree, need to be considered. This provides a nice interpretation of the Generalized Levinson algorithm [FMKL] [FKML]. It is worth mentioning at this point that the Generalized Levinson algorithm for inverting α-Toeplitz matrices may also be derived from the (block) Levinson algorithm applied to a particular $(\alpha + 1) \times (\alpha + 1)$ block Toeplitz matrix [DM1].

The algorithms considered up to now involve first order Schur complements

4.3 Nearly Stationary or Finite Rank Processes

In this section we present summaries of our continuing work on nearly stationary processes. We made significant advances towards a better understanding of such processes and the algorithms related to their estimation. Embedding techniques play a central role in our treatment of nearly stationary process. We were able to show that a large class of problems involving nonstationary processes of a certain type, can be embedded in a multichannel stationary problem. The papers summarized below provide a comprehensive framework for deriving efficient estimation algorithms for these processes. Significant progress was also made towards achieving a stochastic interpretation of the generalized Levinson algorithm. We have a much better understanding of this algorithm and of its different implementations. It is clear now that the time and order update equations of the generalized Levinson algorithm lend themselves naturally to distributed implementation. A more detailed discussion of this point is presented next.

A Tree Classification of Algorithms for Toeplitz and Related Equations [MD1]

The development of VLSI realizations of tree machines, see e.g., [BRO], [SEQ] motivates a study of the structure of fast matrix inversion algorithms from a tree viewpoint, see also [ADM].

An algorithm for efficiently inverting matrices "close to Toeplitz" has been recently proposed in [DGK1]. In that context, an $n \times n$ matrix is considered close to Toeplitz if its contiguous principal submatrices of any given order belong to only a few classes, where two matrices belong to the same class if they are either identical or the reverse of each other. The tree underlying the algorithm is the linear tree associated with the contiguous principal submatrices of order 1 to n. The algorithm performs (one step) *order* updates to invert subma-

4.2 Two-Dimensional Systems

Our earlier studies in 2-D systems have revealed that it is very difficult to generalize most of the essential results for 1-D systems, such as signal processing techniques associated with only time dependent signals. This led us to reexamine the 1-D techniques and, in particular, study very carefully the estimation problems for processes with finite displacement rank, see e.g., [DM1]-[DGMV], since we realized that this theory can be extended to tackle the statistical filtering of non-stationary random fields.

The work of Livshits on operator colligations [L] and, more specifically, on vector colligations [LY] as well as some more recent papers [L2], [KRA] lay the ground to such a generalization. The theory is based on the use of data collected along paths and is in that sense akin to an approach of Willsky et al. [WS]. As a consequence it can be used for estimation not only for random fields on the plane or any Euclidean space but also for instance on the sphere; hence is naturally adapted to geo-location problems.

Although Livshits does not study the estimation problem, he introduces the basic concepts necessary for filtering based on covariance information only. Filtering based on second-order statistics is very appealing from a practical standpoint since usually estimates of higher order moments are not reliable. Furthermore local state-space models, such as Roesser's for 2-D random fields, see e.g., [KLMK], do not need to be introduced. The notions of reflection coefficients and ladder structures for finite rank random fields appear as a key concept from both theoretical and computational viewpoints. An important issue is the determination of the relationship between the reflection coefficients obtained from this approach and the ones introduced by Marzetta [MAR] and Delsarte et al. [DGK2]-[DGK3] in the particular case of stationary 2-D random fields with data collection using line-by-line scanning.

dimensional systems.

We are currently working on developing basic results for multi-dimensional systems which will provide a general framework for handling DSN-type problems. Preliminary research has indicated that there are non-trivial differences between the 2-D and 3-D cases, which makes direct extensions of known results more difficult.

Another fundamental issue that needs to be addressed is the assumption of stationarity, which is a basis for practically all current estimation techniques. The time varying nature of acoustic propagation models and the various events that can occur in a DSN system are highly non-stationary. It is imperative, therefore, to develop estimation techniques capable of handling non-stationary processes. Using the notion of nearly stationarity, we obtained a useful characterization of non-stationary processes, that lends itself to analysis and algorithm development. Significant advances have been made in our understanding of non-stationarity and a number of interesting results were obtained. Another important aspect of our work on non-stationary processes is related to the possibility of their implementation by distributed algorithms. Our analytical framework turns out to provide a natural problem decomposition, that can be utilized for distributing the required computations among multiple processors.

(phase shifts, FFT-s) fall into this category.

The special structure of the source location problem, in a homogeneous propagation media, and the assumption of Gaussian statistics makes possible the separation of spatial and temporal estimation. However, this separation fails for more complicated propagation models. Even in relatively simple cases, this type of approach is not completely satisfactory where multiple sources are present.

Our approach has therefore been to consider a multi-dimensional estimation problem. In fact, when only a finite number of discrete sensors are used, the problem is multi-channel, rather than multi-dimensional.

The system consisting of several sources and sensors can be viewed as a multi-input multi-output (MIMO) system. Under certain assumptions on the source spectra and the propagation model, the vector time-series observed by the sensors can be modeled as a multi-channel ARMA process. By using various least squares or maximum likelihood parameter estimation techniques, it is possible to fit a set of ARMA coefficients to the measured data. We have been able to show that source locations and their spectra can be obtained relatively easily from these coefficients. In general, there exists a mapping between the properties of the source location problem into the structure of the ARMA coefficients. The details of this mapping and the structure it imposes on the MIMO system is presently under investigation.

As mentioned above, multichannel estimation is a special case of multidimensional estimation. A large body of results is currently available for 2-D signal processing. These results were developed mainly in the context of image processing, where two spatial dimensions are considered. Relatively little seems to have been done in the case where one spatial dimension and one temporal dimension are involved. Even less seems to be known about estimation in higher

4. BASIC RESEARCH ON DISTRIBUTED ALGORITHMS

4.1 Overview

The analysis and design of a distributed sensor network involves a variety of issues. Very little basic research has been done on the fundamental mathematical problems that need to be solved in order to provide a systematic approach to DSN system design. While most of our work concentrated on the more immediate problem of developing signal processing tools for the DSN, we devoted part of our effort to more basic research. In this section we discuss two research areas of this kind: multi-dimensional estimation, and nearly stationary processes.

Multiple sensor measurements can be considered as samples of a time-space function $y(t, \mathbf{x}; p)$ generated by the source, where $\mathbf{x}$ represents the sensor location in 3-D space, and p is a vector of source parameters. This function may represent electromagnetic waves reflected from an object or acoustic waves generated by aircraft engines. The parameter vector p may include source location, velocity, and spectral characteristics. Thus the problem is to estimate the unknown parameter vector p from a set of noisy measurements.

Current approaches to this problem are almost exclusively based on reducing the multi-dimensional estimation problem to a sequence of one dimensional problems. Typical array processing techniques involve forming multiple beams by performing linear operations on the sensor outputs. This linear operation reduces the dimension of the problem by effectively concentrating on a single spatial direction at a time. The ensuing signal processing involves a scalar time series and thus standard estimation and filtering techniques can be applied. All the different versions of beam-forming and TDOA estimation techniques, whether implemented in the time domain (delays, correlators) or in the spectral domain

Efficient Solution of Lyapunov Equations [PM], [HM]

The Levinson algorithm for fitting an autoregressive (AR) model to a given covariance matrix, has many nice properties. In addition to being computationally efficient is provides not only a set of AR coefficients but also the related reflection coefficients and the backwards predictor coefficients. In fact, the Levinson algorithm yield two Cholesky factorizations of the inverse covariance matrix. In many applications we encounter the inverse problem: given a set of AR coefficients, find the covariance sequence, or the reflection coefficients. In DSN applications the problem arises in the context of translating "standard" parameters (i.e., parameters of models given in direct form) into the ladder parameters required by our algorithms. A similar problem is encountered when transforming left to right MFD's.

The solution of the inverse problem is relatively easy in the scalar case. The reason is that the backward predictor is then just the reversed order forward predictor. Therefore, it is possible to run the Levinson algorithm backwards starting from the given full-order solution and going back to zero order. Unfortunately, this procedure does not carry through to the matrix case, since the backward predictor is not longer a simple function of the forward predictor coefficients. Finding the covariance sequence requires solution of the discrete Lyapunov equation. Straightforward solutions of this equation are computationally expensive for high order systems. In [PM] and [HM] we develop novel solution techniques that utilize the problem structure to significantly reduce computational requirements. Three efficient algorithms are presented. One of the methods is based on a recently described method of inverting matrices that are sums of block-Toeplitz and block-Hankel matrices [FM3]. The procedures are then shown to yield a stability test for the given autoregressive model.

By properly choosing the delay D (or the pre-filter), the wide band components at the two inputs become decorrelated while the narrow band signal remain correlated. Since the coefficients of the filter are updated to minimize the error power, the predictor and error outputs can be applied to line enhancement and signal whitening respectively. Most analyses of the ALE were for a white noise background, while very few results exist for the colored noise and the whitening performance. Satorius and Zeidler considered an autoregressive moving average (ARMA) input process [SZ].

In [NM], a matrix formulation was used to derive the optimal coefficients and frequency response of the ALE and exact least-squares predictors, for inputs of real or complex sinusoids in general additive colored noise. The derivation of the amplitude gains of the output sinusoids generalized known results for the white noise case. In low-pass and high-pass background noise the amplitude gains become essentially monotonically increasing and decreasing functions respectively of the sinusoid frequencies. The amplitude distortion that is introduced by the correlated noise can be reduced by choosing a larger number of coefficients, L. In evaluation of the performance of the predictors in whitening applications, an upper bound on the output SNR was found when filtering a white signal that had been correlated by multiple stationary sinusoids.

To enable filtering of nonstationary complex and multichannel data, a complex vector version of the ladder algorithm is developed. It can be used to implement the complex ALE, as well as the noise canceller or noise inversion. A major advantages of this algorithm is its rapid rate of convergence to the exact least-squares solution, hence it can actually achieve the upper bounds of the SNR performance.

used: some fraction of the alpha pseudo innovations is actually passed through anti-causal or adjoint filters instead of causal ones. This mixed representation avoids a particular signature problem. Physically, the necessary algebra is very much reminiscent of modern physics; the different pseudo processes have similar properties as particles and anti-particles, in the sense that they can be converted from real causal/anti-causal representations to imaginary anti-causal/causal representations. Other evidence, such as the fact that Livshits definition of the (displacement) rank of a process generalizes to multi-dimensions, makes the physical insight even more interesting. The implications of these results to continuous time models and the solution of Fredholm integral equations are outlined.

As a last topic we mention the direct identification of alpha stationary processes. Because of the multiplicity issue the question of degree of freedom of models for such processes has to be raised. A direct answer would probably be quite difficult; however, a different parametrization of the models using so-called ladder or lattice form realizations, see e.g. [MLNV], [ML2], [LEE], lead to a very promising alternative. We only note here that the gaussian log-likelihood functions and their derivatives with respect to the model parameters are simple functions that can be straight forwardly analysed.

Distortion Measures of Finite Rank Processes Via Ladder Forms [LM4]

It has been known for some time that distortion measures of second order processes can be evaluated using state-estimation and related techniques. Measures such as the Bhattacharyya, the Kullback-Leibler, and the related Itakura-Saito for second order processes involve determinants, traces and ratios of covariance matrices, see e.g., [KA3]. In many applications, it is of interest to compute these measures efficiently. In speech processing, these measures are important, for instance, in the design of low rate speech encoder [MBG], as well as for speech recognition, both for the discrete utterance [ITA], and continuous speech cases [JEL]. In pattern recognition, these measures are important in feature extraction and clustering analysis. In digital communication, these measures arise in channel capacity calculations.

In many applications, one may be interested in efficient parametrizations of such measures, either to simplify the calculations and design, or in the hardware implementation. Efficiency is of interest if the optimization of such measures has to be done "on-line" or "adaptively". Such an optimization has become feasible, due to the availability of very large scale integrated circuits (VLSI).

It is well known that distortion measures of stationary processes involve covariance matrices and kernels of the Toeplitz or displacement type. These measures require either the solution of linear equations involving such operators, or the computation of triangular factors or volterra kernels. They can be obtained efficiently via the use of existing methods, such as the Krein-Levinson type algorithms.

The first aim of this paper is to emphasize that the stationarity is not necessary for obtaining computational benefits and ease of parametrization; that is, we are now able to efficiently compute distortion measures for processes that deviate from stationarity in a certain sense. An index of stationarity, called "shifter low rank" (displaced) rank of alpha, has recently been introduced, [Mo],[FMKL] [KLM] [KKM], with the significance that the larger the value of alpha the more non-stationary a process is. In addition, the complexity of process models and calculations involving such processes increases only proportional to this index. In the discrete case, alpha can be used to measure the distance of a matrix, e.g., a covariance matrix, from being Toeplitz. Our earlier results showed that Krein-Levinson and other algorithms for Toeplitz operators can be generalized to such alpha-stationary processes, as well as a new, and even more efficient doubling type algorithms [M3].

Secondly, we show that ladder canonical forms, which are realizations of processes based on partial-correlation coefficients, have many advantages in computing distortion measures. These ladder forms in fact represent a "natural" canonical parametrization of many of the measures, in the sense that the measures become simple functions of the ladder form parameters. They typically involve weighted averages, including logarithms. For instance the gaussian log-likelihood derivative with respect to these ladder parameters is a linear function in these parameters! Moreover, the ladder forms have many other nice features, such as stability and automatic scaling properties.

Several examples involving distortion measures are given, and their physical significance and applications to communication and estimation, e.g. speech processing and digital communication is discussed.

Efficient Inversion Formulas for Sums of Products of Toeplitz and Hankel Matrices [FM3]

The problem of solving linear equations, or equivalently of inverting matrices, arises in many fields. Many techniques for doing this have been developed, but all of them have the characteristic that it takes in the order of N^3 operations (multiplications and additions) to invert a general N x N matrix. A great deal of work has been devoted to finding more efficient ways of inverting matrices, by utilizing any special structure that they may posses. This special structure can take various forms, but there are many problems in mathematical physics and statistics in which the matrices can be shown to be Toeplitz or block Toeplitz, i.e.,

$$R = [r_{(i-j)}] \ , \quad 0 \le i,j \le N \ , \tag{4.1}$$

where r is a p x p block entry. Some examples for applications involving Toeplitz matrices are least squares estimation problems involving convolutions and integral equations with a difference kernel. Hankel matrices represent another type of special structure, characterized by

$$R = [r_{(i+j)}] \ , \quad 0 \le i,j \le N \ . \tag{4.2}$$

Several recursive algorithms have been developed for the inversion of Toeplitz and Hankel matrices, requiring only $O(N^2)$ arithmetic operations. The extension of this work to block-Toeplitz and block-Hankel matrices of size Np x Np (i.e., N x N blocks of size p x p), shows that the inverse can be obtained in $O(p^3N^2)$. Recently some new algorithms have been developed that compute the inverse of a Toeplitz matrix even faster, in $O(N\log^2 N)$ operations.

While Toeplitz and Hankel matrices appear in numerous applications, it often happens that practical problems involve matrices with a more complex structure. As an example, the inverse of a Toeplitz matrix is no longer Toeplitz but it retains a very simple structure. In previous work we have shown that efficient inversion formulas can be obtained for matrices that are "close to Toeplitz" in some sense. More precisely, we considered the class of matrices represented by

$$T^{\alpha} = T + \sum_{i=1}^{\alpha} \sigma_i L_i U_i \ , \tag{4.3}$$

where

T = full Toeplitz matrix

$L_i(U_i)$ = lower (upper) triangular Toeplitz matrix,

$\sigma_i = \pm 1$

and α is the smallest integer for which this decomposition holds. We have shown that the number α associated with a given matrix is unique, and given by

$$\alpha = rank\ \delta[T^{\alpha}] \tag{4.4}$$

where $\delta[\cdot]$ is the shifted-difference operator given by

$$\delta[R] = \begin{bmatrix} r_{1,1} & \cdots & r_{1,N} \\ \vdots & & \vdots \\ r_{N,1} & \cdots & r_{N,N} \end{bmatrix} - \begin{bmatrix} r_{0,0} & \cdots & r_{0,N-1} \\ \vdots & & \vdots \\ r_{N-1,0} & \cdots & r_{N-1,N-1} \end{bmatrix}, \tag{4.5a}$$

$$R = [r_{i,j}]\ , \quad 0 \le i,j \le N\ . \tag{4.5b}$$

We will refer to a matrix of this type as having a shift-low rank or displacement rank of α, or more briefly, as an α-Toeplitz matrix. This class of matrices has a nice closure property in the sense that the inverse of α-Toeplitz matrix is also α-Toeplitz.

The natural extension of these results to matrices that are "close to Hankel," involves the following type of representation:

$$H^{\alpha} = \overline{T}\tilde{I} + \sum_{i=1}^{\alpha} \overline{\sigma}_i\, \overline{L}_i\, \overline{U}_i\, \tilde{I} = T^{\alpha}\tilde{I} = \tilde{T}^{\alpha}\ , \tag{4.6}$$

where $\overline{T}, \overline{L}, \overline{U}, \overline{\sigma}$ are defined in the same way as T, L, U, σ. In other words, the α-Hankel matrix is simply a column-permuted version of an α-Toeplitz matrix. Thus, a trivial way of inverting a α-Hankel matrix is by transforming it into an α-Toeplitz matrix and then applying the algorithm given for instance in [FMKL], i.e.,

$$[H^{\alpha}]^{-1} = \left[\overline{T} + \sum_{i=1}^{\alpha} \overline{\sigma}_i\, \overline{L}_i\, \overline{U}_i\right]^{-1} \tilde{I}\ . \tag{4.7}$$

However, it is also possible to invert the α-Hankel matrix directly. The difference between these two types of inversion formulas lies in the sequence of submatrices that are recursively inverted. This subtle difference makes the trivial solution (4.7) inapplicable in many situations, e.g., if $H^a = H^u \oplus H^l$.

In [FM3] we extend the class of matrices for which efficient inversion is possible, to include mixtures of Toeplitz and Hankel matrices. We consider matrices of the form

$$R = T + \sum_{i=1}^{\alpha} a_i L_i U_i + \tilde{T} \tilde{I} + \sum_{i=1}^{\beta} \tilde{a}_i L_i \tilde{U}_i \tilde{I} = T^{\alpha} + H^{\beta} , \qquad (4.8)$$

i.e., sums of α-Toeplitz and β-Toeplitz matrices. This class of matrices has a closure property in the sense that R^{-1} has a representation of the type given in Equation (4.8).

The basic idea underlying our results, is to embed the α-Toeplitz/β-Hankel matrix in a larger matrix that has a structure for which efficient inversion formulas have already been developed. Two such embeddings are explored: We first show that an α-Toeplitz/β-Hankel matrix R with p x p entries (i.e., $R = [r_{i,j}]$, $r_{i,j}$ is a p x p matrix), can be embedded in an $(\alpha+\beta)$-Toeplitz matrix with 2p x 2p blocks. Applying the inversion formulas derived in [FMKL] to this matrix results in an algorithm which requires $O(4(\alpha + \beta + 2)p^2N^2)$ operations. Second we present a way of embedding the α-Toeplitz/β-Hankel matrix in a block Toeplitz matrix with $2(\alpha + \beta + 2)p \times 2(\alpha + \beta \;\; 2)p$ entries. Applying the multichannel Levinson algorithm to this matrix requires $0(8(\alpha + \beta + 2)^3 p^3 N^2)$ operations. However, by using the special structure of the block entries the algorithm can be further simplified, resulting in an algorithm equivalent to the one mentioned before. Further reductions in computational requirements can be obtained by applying the class of doubling formulas (i.e., the $O(N \log^2 N)$ algorithms described in [GY] [AHV] [M3] to the embedded block Toeplitz matrix.

5. DEVELOPMENT of SIGNAL PROCESSING FACILITIES

Signal Processing Workbench

An essential part in the process of developing new techniques for source parameter estimation is the testing of proposed algorithms by computer simulation. Simulations allow us to study various issues related to the performance of the algorithms and to gain insights into their behavior. Analysis alone is not adequate, especially when recursive stochastic algorithms are concerned. Simulations are an invaluable tool for studying issues such as convergence rates, robustness, and estimation accuracy. Therefore, establishment of a signal processing workbench that will allow easy comparison and modification of signal processing steps and input data sequences in important. In particular, this will facilitate the exchange of processing tools and data bases among DSN researchers, and maximize the impact of our research results on the design of a DSN system.

In the proposed workbench, the processing modules function in a standalone fashion with standard input/output formats such that complex processing functions are built by the simple cascading of modules. In the UNIX operating system, precompiled stand alone modules can be connected together using a system command called a pipe. Thus signal processing can be changed quickly without recompiling or linking, just forking to the appropriate module. The signal processing workbench should readily allow inputs derived from actual experiments or simulated inputs and allow for their perturbation by various types of noise.

Observing and modifying the internal parameters of a signal processing procedure while data is being processed greatly enhances the debugging and understanding of signal processing algorithms. The version 7 UNIX operating system for the VAX computer incorporates a debugger that allows the internal parameters of a signal processing submodule to be observed and modified as the data is being processed. The current UNIX debugger applies only to procedures within a main program. For cascaded stand alone programs, this facility must be extended. Some experience with the problem of adding this capability has been obtained on the DEC-11/34 through the development of our own debugger. This experience will be applied to extensions of the debugger for the VAX.

In several theoretical and applied problems we are running up against a boundary characterized by the need of manipulating formulas. This raises the urgent need for obtaining software to handle algebraic formulas symbolically. We made an effort to obtain a copy of MACSYMA for this purpose; however, we were unsuccessful so far, due to lack of cooperation from the originators of MACSYMA.

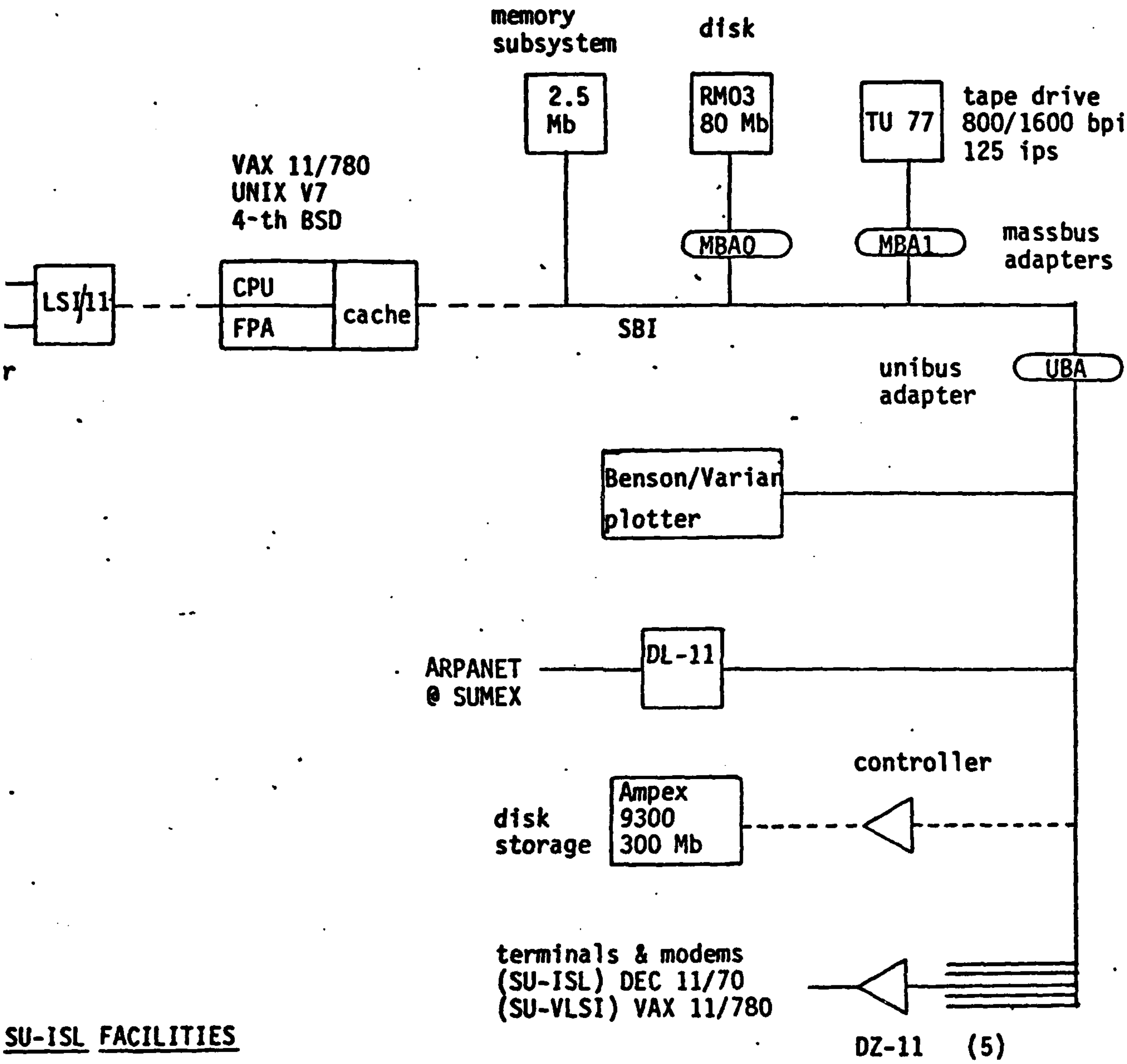
DSN Computer Facility

The recently expanded computer facility for investigating DSN strategies is centered around a Digital Equipment Corporation VAX 11/780, see Figure 5.1, running under the UNIX operating system. The installed operating system is the latest available from the Electrical Engineering and Computer Science Department of the University of California at Berkeley, the Fourth Berkeley Software Distribution (4BSC) of UNIX version 7, released November 1980. Our current VAX computer hardware incorporates a 2.5 Megabyte main memory subsystem and 80 Megabytes of disk storage, provided by a single RM03 disk drive. This storage is to be augmented with an Ampex 300 MB disk drive by early 1981. Our computer is interconnected with a DEC 11/70 (SU-ISL) and a VAX 11/780 (SU-Mount St. Helens) and to the ARPANET. The SU-DSN VAX became operational in December 1980 and the SU-ISL 11/70 was brought up in November 1980, replacng a much overworked DEC 11/34 system. The 11/70 is running UNIX version 6. nstalled on the VAX is the new RAND text page editor (called E) which gives the ıser powerful functions for moving text within a file and provides greatly mproved recovery of working data sets after system crashes.

In the last two years prior to the acquisition of the VAX and 11/70, several computers were used to test algorithms and to evaluate their performance under simulated conditions. Among these facilities used were the ISL 11/34, the VAX (Mt. St. Helens) which became operational on April 1980, and the facilities of the Stanford Artificial Intelligence Laboratory.

The additional 300 MByte disk drive, to be delivered in the spring of 1981, is necessary to handle the quantity of data expected in the typical DSN scenario. Further establishment of an interactive graphics system will enhance our work on application of image reconstruction techniques to the DSN problem. Two high resolution bit map display systems are being added to our VAX, one can be upgraded to gray scale and color later. We have ordered a laser printer to upgrade the quality of text and hardcopy graphics output. In order to efficiently perform image reconstruction and target tracking using data from multiple sensor nodes, our fast access memory should be extended.

a) SU-DSN:



SU-ISL FACILITIES

DEC 11/70 375kb CPU memory & FPU-A
2 Ampex 9300 300Mb disks
Digidata tape drive 800/1600 bpi 45 ips
Printronix line printer 300lpm
2 Diablo 1640 terminals
Tektronix 4013 graphics terminal
dial out modems

terminal interconnect facility to SU-DSN, SU-ISL, SU-MT. ST. HELENS and other micro-computer systems

Figure 5.1 SU-DSN COMPUTER FACILITIES

are and Hardware Programming Efforts

Our early efforts in software development were aimed at establishing a pro- library of system identification and parameter estimation procedures. library includes standard recursive identification methods, prewindowed r algorithms in both AR and ARMA forms, and newly developed covariance sliding window ladder form algorithms. Normalized and unnormalized ver- of the algorithms have been written. The ladder form algorithms were en primarily in "C" while the other routines were written in Fortran or SAIL, the language of the Stanford Artificial Intelligence Laboratory.

A speech modeling system was previously developed around the prewin- d ladder form as described in [ML1]. The reflection coefficients which meterize the speech spectrum were determined recursively with updates y new speech sample. At the desired transmission rate, the best single e of a reflection coefficient is chosen. The likelihood parameter from the ction coefficient updating equation was used to identify the pitch pulse loca- . Event detection algorithms that indicate a sudden change in the underly- ignal structure evolved from the pitch extraction technique.

The prewindowed ladder form has also been the basis of an investigation efficient hardware implementations using the CORDIC computation tech- e to reduce the complexity of the algorithm. The study led to a series of chip designs, that turned out to be extremely promising for general real digital signal processing. A data modem using the ladder form was also nined.

The linear equation technique for solving the source location problem from difference of arrival data has been verified by computer simulation. The ts of imprecise sensor location and noisy TDOA information have also been ılated.

Work on modification of medical image reconstruction techniques for the DSN scenario is continuing. The minimum variance approach to Image Reconstruction developed by Wood [WOO], Fortes [FOR] and Nunes [NUN] originally was programmed to process phantom image of size 9 pels by 9 pels because of the limited computer main memory available at the time. With our recently enhanced computer facilities, these techniques can be extended to a larger size of sensor array that would occur in the DSN scenario.

Relations with other DSN Contractors

In order to facilitate experimentation with ladder form estimation algorithms on actual acoustical sensor data and to take advantage of existing DSN software, we are establishing the Lincoln Laboratories DSN signal processing software at Stanford. Their software package processes acoustical data recorded on digital tape from their microphone arrays during aircraft flyby experiments. The processing of data from a single sensor array produces a power versus azimuth plot at various temporal frequencies and elevations of interest. Appropriate interpretation of this data yields bearing information for sound sources which can be amalgamated into tracks of possible target motion. The Lincoln Lab package produces only bearing information. Since the maximum separation between sensors in the current array is three meters, it is doubtful that high resolution range information for distant sources could also be obtained from the current single array data acquisition system. Using the data tape reading and file I/O structure of this package, we are extending the power versus azimuth calculations to include ladder form spectral estimation techniques.

Our efforts to establish a working system here, first on a VAX and later on a 11/70 have been frustrated by many difficulties which further point to the ity o. establishing a common signal processing workbench. The software ge written in "C" for a 11/70 running UNIX version 7 is quite large and es several software libraries written by several groups at Lincoln Labs. ierable time was spent in locating all the necessary source files and tly transferring them over the ARPANET initially to the Mt. St. Helens VAX iter. The location of the various source components is listed below. The is for the Tektronix plotting routines came from the Lincoln Laboratories d Seismology group.

ource	/u0/green/dsnt
eclarations and shell files	/usr/bin
rary -list management system	/usr/src/lms
rary -parameter data base subroutine	/usr/src/pdbsubs
'ary -plotting routine sources	/rm13/crames/graph
from ll-asg	/gpactek

he attempt to compile and run the package on the VAX revealed a number tability problems, including illegal operations on pointers and incompati- e of data types. Manipulation of the data structures and file header infor- n and definition of file I/O variables were based on an assumed (11/70 t) 2 byte integer word length. Since the VAX word length is 4 bytes, this loes not work on the VAX. This use of machine dependent code appeared itly so that conversions were substantially more involved than redeclaring le types. The large size and monolithic construction of the package made nsequences of these problems severe. Testing for program bugs was com- ed by the many layers of subroutine calls, the probability of multiple bugs, e sheer volume of code to search. Since changes to one procedure tended causes a need for modifications in other procedures, fixing bugs was a tedious peration. Even when changes were confined to one procedure, the modification rocess was slow because of the need to reload the entire package. After making a list of the problems and discussing them with the group at Lincoln Labs, we eached an agreement that they would rewrite the package so that it would be ompiler warning free, not have the word length problems, and be compatible ith the VAX computer.

The recent acquisition of a 11/70 computer by ISL provided the opportunity attempt to bring up the original Lincoln Laboratory package on the computer r which it was written. However our system was running UNIX version 6 rather ian the version 7 of Lincoln Labs. We experienced problems with several iodules that were in the version 7 operating system that did not exist in version or would not run properly. These include the matrix space allocation routine nd the ordered library building routine. The large number of source files aused problems in compiling and loading the program. The list of file names xceeded the maximum number of parameters for a load command, so libraries ad to be constructed. Due to the large program size and the restricted iemory of the DEC-11/70, the program and data areas have to be separated. e currently have the azimuth analysis portion of the Lincoln Labs software running but not the plotting software. Using a test data tape of artificial data, we hink that our version of the software is functioning identically to the Lincoln abs software. We are awaiting a data tape of aircraft flyby experiments in rder to apply our ladder form algorithms on real sensor data.

Several recommendations can be made, based on the experience of tranporting this package:

Portability should be a high priority in selection of programming style. This will facilitate both transfer of software between groups and transfer of

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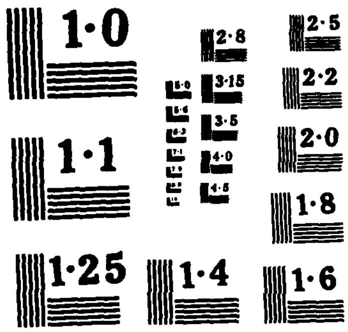

NATIONAL BUREAU OF STANDARDS
MICROCOPY RESOLUTION TEST CHART

to causes a need for modifications in other procedures, fixing bugs was a tedious operation. Even when changes were confined to one procedure, the modification process was slow because of the need to reload the entire package. After making a list of the problems and discussing them with the group at Lincoln Labs, we reached an agreement that they would rewrite the package so that it would be compiler warning free, not have the word length problems, and be compatible with the VAX computer.

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Several recommendations can be made, based on the experience of transporting this package:

- *Portability* should be a high priority in selection of programming style. This will facilitate both transfer of software between groups and transfer of software to new machines in the same group.

- *Installing* and *maintaining* software is easier when programs are *small and simple* in structure.
- Complex signal processing functions should be achieved through *linking of simple modules*, rather than by building large, complex programs.
- Last but not least, a *real-time* version of UNIX should be developed.

6. CONCLUSIONS

We have made considerable progress towards the development of new algorithms for the various signal processing functions acquired in a DSN system. Our theoretical research on the source location problem and on estimation techniques has reached a fairly mature stage. We now need to turn these results into software modules that will be usable by Lincoln Laboratory and other members of the DSN community. The signal processing workbench will enable us to test the algorithms we have already developed, as well as to refine them and discover new ones. We need to evaluate the performance of these algorithms using both synthetic and real data.

Our goal is to gradually replace the signal processing modules currently used by Lincoln Laboratory with our own improved modules. This will be done in stages. First, some of the basic components will be replaced, but the overall processing structure will remain unchanged. For example a high resolution spectral estimator based on the FFT is now used as the front end for source location detection. We can replace this estimator by one based on ladder forms, which is expected to have superior performance. In a second stage we may want to make more comprehensive changes, which may require structural modifications. For example, a combination of a multichannel ladder form with an event detection algorithm may replace the current detection scheme. A combination of an ARMA modeling technique for TDOA estimation and the new algorithms for source location described in Section II may replace the current localization technique. See also [DSN] for an overview of the signal processing architecture associated with these new algorithms. This process of replacing processing modules in a prototype DSN system will provide a conclusive test of our algorithms, and lead the way to their widespread use in multi-source tracking/classification applications. A test of this type is an essential step in the development of any new technique.

The signal processing workbench will play a central role in the developments described above. It will enable us not only to test and exercise our algorithms, but will be used as a valuable research tool. Some of the algorithmic work is heavily dependent on simulations because it is not analytical. Also, other sophisticated programming languages (e.g., MACSYMA) will be needed to handle the complexity of more sophisticated algorithms. So far we treated mainly one dimensional problems. The development of two and three dimensional estimation algorithms is expected to involve complex equation manipulations that are too hard to perform manually. We feel that our progress in this area will be limited without computer aided equation handling.

In the next phase of our project, we also plan to emphasize research into some of the fundamental mathematical issues that need to be resolved to achieve advanced DSN system design. Much of the earlier work on multi-source, multi-sensor signal processing involved relatively simple modeling (single channel, stationary processes, etc.). There is a clear need to develop more sophisticated mathematical models and the analytical tools related to them. Some examples of open research issues are briefly mentioned below:

(i) Performance bounds on tracking classification algorithms, given multi-sensor multi-source information, assuming non-stationary and non-gaussian processes. Such bounds are essential for the evaluation of the performance of any DSN system, i.e., how far away from "optimal" is it ?

(ii) Additional insights are needed into the question of distributed processing. We need to find ways of characterizing the minimal information that needs to be exchanged in a distributed system in order to achieve a given performance level. How much of the information can be used only at the local level, and how much needs to be communicated to other nodes.

(iii) Development of efficient estimation algorithms for two and three dimensional systems. We need to find a good spectral representation for multidimensional processes. It seems clear that there are differences at a very fundamental level between the structure of multi-dimensional and one dimensional problems. As mentioned before, the complexity of the equations arising in such systems makes it difficult to search for structural patterns.

(iv) A different approach to distributed computations, that are not explored so far, is to use probabilistic algorithms of the type developed by Rabin. Independent solutions by multiple nodes followed by majority voting can lead to a natural way of distributing computations in a complex algorithm.

(v) A large class of DSN problems combines both processing and communication of data. The communication aspects of these problems involve the theory of multiterminal communication. The main results available so far show bounds on the channel capacity (the maximum rate of information transmission) for broadcast or multi-access-type channels. Most of these results are of the existence type, i.e., do not provide practical schemes that could be implemented. See [VM] [BER] [AEG].

In summary, we plan to continue our investigation on two levels, basic research into fundamental mathematical issues of the DSN, and the development/implementation/testing of signal processing modules for the prototype DSN system. This approach combines long term goals such as optimal DSN design methodology with short term performance gains.

7. DSN PUBLICATIONS

This section summarizes the publications and presentations resulting from our work on the DSN project. With few exceptions, these publications are described in Sections 2, 3 and 4.

SUBMITTED:

B. Porat, M. Morf and D. Morgan, "On the Relationship Among Several Normalized Square-Root Ladder Algorithms," *CISS*, submitted January 1981.

A. Nehorai and M. Morf, "Enhancement of Sinusoids in Colored Noise and Whitening Performance of Least-Squares Predictors," *IEEE Trans. ASSP*, submitted January 1981.

B. Porat, B. Friedlander and M. Morf, "Square Root Covariance Ladder Algorithms," *IEEE Trans on Automatic Control*, submitted December 1980.

B. Egardt and M. Morf, "Asymptotic Analysis of A Ladder Algorithm for ARMA Models," *IEEE Trans. on Automatic Control*, submitted November 1980.

M. Morf, "Displacement Ranks and Doubling Algorithms for Matrix Inversion," *Bulletin of the American Mathematical Society*. submitted 1979.

M. Morf, "Doubling Algorithms for Toeplitz and Related Equations," *IEEE Trans. Inf. Theory*, to appear 1981.

R. Longchamp and M. Morf, "Distributed Gaussian Second-Order Filters," *IEEE Trans. on Automatic Control*. submitted December 1979.

M. Morf, "Extended System and Transfer Function Matrices and System Equivalence," *Int. J. of Control*. submitted November 1979.

M. Morf, "Fast Cholesky Algorithm and Adaptive Feedback Ladder Forms," *IEEE Proc. 20th CDC*, Dec. 1981, submitted March 1981.

P.R.R.L. Nunes and M. Morf, "Image Reconstruction Techniques and Target Detection," *IEEE Proc. 20th CDC*, Dec. 1981, submitted March 1981.

D.T.L. Lee, J.-M. Delosme and M. Morf, "State-Space Structures of Generalized Ladder Canonical Forms," *IEEE Proc. 20th CDC*, Dec. 1981, submitted March 1981.

J.-M. Delosme and M. Morf, "Normalized Doubling Algorithms for Finite Rank Processes," *IEEE Proc. 20th CDC*, Dec. 1981, submitted March 1981.

H. M. Ahmed and M. Morf, "On the Relationship Among Certain Numerical Algorithm and Bilinear Bang-Bang Control," *IEEE Proc. 20th CDC*, Dec. 1981, submitted March 1981.

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M. Morf and A. Nehorai, "A New Algorithm for Recursive Estimation of Right Matrix Fraction Description Type Multivariable Systems," *IEEE Proc. 20th CDC*, Dec. 1981, submitted March 1981.

Mohamed T. Hadidi and M. Morf, "Efficient Solution of the Inverse Levinson Problem in the Multichannel Case Via the Lyapunov Equation," *IEEE Proc. 20th CDC*, Dec. 1981, submitted March 1981.

ACCEPTED:

H.M. Ahmed, P.H. Ang and M. Morf, "A VLSI Speech Analysis Chip Set Utilizing Coordinate Rotation Arithmetic," ICCAS, Chicago, IL, April 1981. (invited)

B. Porat, B. Friedlander and M. Morf, "Square Root Covariance Ladder Algorithms," *Proc. 1981 ICASSP*, Atlanta, GA, March 30, 1981, pp. 877-880.

B. Friedlander and M. Morf, "Efficient Algorithms for Adaptive Linear-Phase Filtering," *Proc. 1981 ICASSP*, Atlanta, GA, March 30, 1981, pp. 247-250.

A. Nehorai and M. Morf, "Enhancement of Sinusoids in Colored Noise and Whitening Performance of Least-Squares Predictors," *Proc. 1981 ICASSP*, Atlanta, GA, March 30, 1981, pp. 275-278.

M. Morf, C. Muravchik and D.T. Lee, "Hilbert Space Array Methods for Alpha-Stationary Process Estimation and Ladder Realizations for Speech and Adaptive Signal Processing," *Proc. 1981 ICASSP*, Atlanta, GA, March 30, 1981, pp. 856-859.

H.M. Ahmed, M. Morf, D.T. Lee and P. Ang, "A VLSI Speech Analysis Chip Set Based on Square-Root Normalized Ladder Forms," *Proc. 1981 ICASSP*, Atlanta, GA, March 30, 1981, pp. 648-653.

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